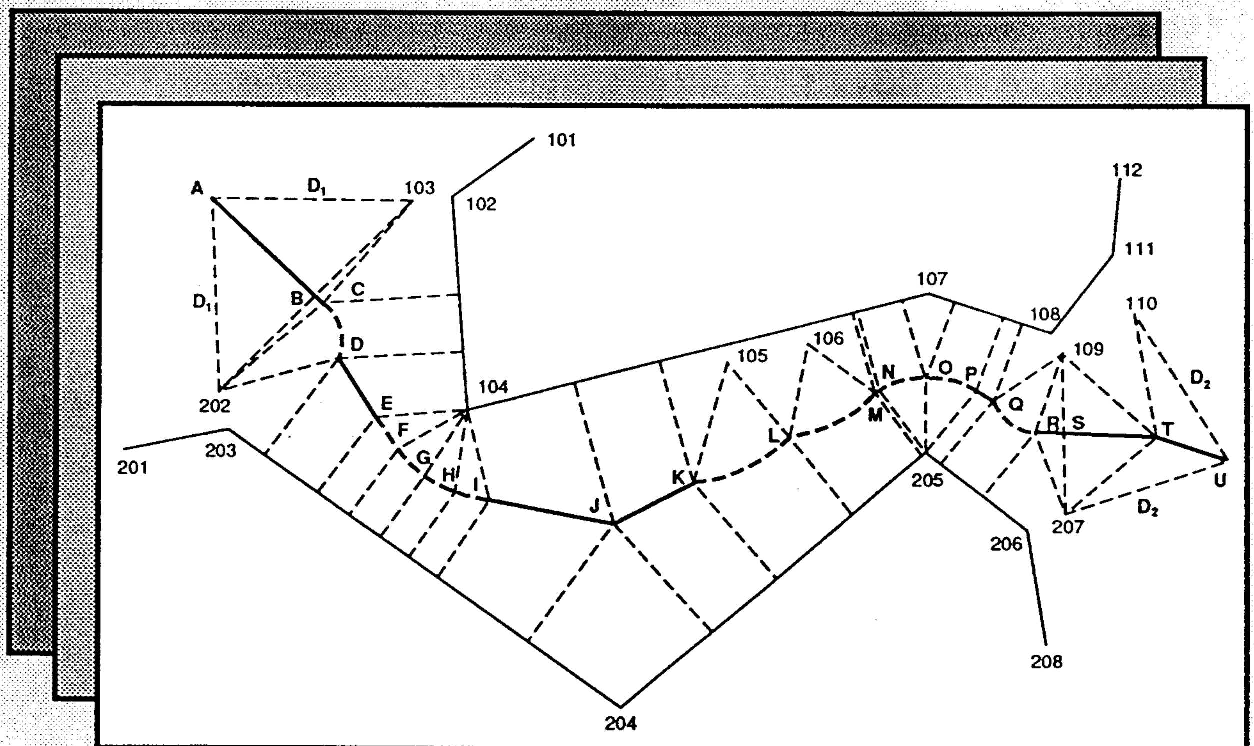
Planar Median (Equidistant) Line Computations for Narrow Channels



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PLANAR MEDIAN (EQUIDISTANT) LINE COMPUTATIONS FOR NARROW CHANNELS

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ABSTRACT

The geometric median line of a body of water consists of a series of intersecting curved line segments and/or straight line segments every point of which is equidistant from the closest points on opposite shorelines.

Shorelines are sometimes defined by meander traverses and are sometimes defined by prominently located salient points only, or by combinations of meander traverses and salient points. The shape and location of a median line is determined by the shape and location of the shoreline, and the type of median line is determined by the type of shoreline definition.

Analytical procedures and mathematical equations are presented that can be used to compute the exact coordinates of the end points, angle points, and points of intersection of all line segments that form a planar median line for all types of shorelines. In addition, analytical procedures and mathematical equations are presented that can be used to compute the coordinates of other important median line points and associated shoreline points which might also be needed to determine the accurate location of, or to write a clear and complete legal description of a median line.

INTRODUCTION

A geometric median line, lateral line, or equidistant line consists of a series of intersecting curved line segments and/or straight line segments, every point of which is equidistant from the closest points on opposite shorelines. In the case of a median line the opposite shorelines are more or less parallel with each other, like the banks of a river, and the median line is located at midpoint between the two shorelines. In the case of a lateral line one shoreline is actually a continuation of the other, like the seashore. The "opposite" shorelines are actually adjacent or side by side, and the lateral line extends seaward following a course that is more or less perpendicular to the seashore. Lateral lines and median lines can be described collectively as equidistant lines.

The term median line is used herein. However, because the median line computational procedures described also apply to lateral lines, the term equidistant line might be more descriptive.

A median line is continuous in the sense that there are no gaps. However, because median line direction changes might sometimes occur abruptly at angle points, a median line might not be everywhere continuous in a mathematical sense.

The procedures outlined for computing the locations of median lines can best be described as semianalytical. The approximate locations and types of median line segments are determined by examining a plot of shoreline points and lines, and the corresponding shoreline/median line computational options are identified. The exact coordinates of median line angle points, end points, points of intersection and other associated points are then computed using the equations presented. A computer program has also been developed, of course, which can be used to compute all needed coordinates after the most appropriate shoreline/median line computational options have been identified. The most frequently encountered shoreline/median line combinations are illustrated in Fig. 1 and are described on pages 4 through 10. The descriptions and the Fig. 1 diagrams are linked by the computational-option abbreviations, such as [PIL2], for example, that appear with both.

Two different approaches can be applied to determine the coordinates of the points of intersection of adjoining median line segments. In the first approach the types and locations of the median line segments are determined first, and the coordinates of the points of intersection of adjoining segments are then computed separately. In the second approach all appropriate geometric criteria are utilized to derive a set of equations which are then solved simultaneously to obtain the coordinates of the points of intersection of adjoining median line segments and the coordinates of other median line defining points (points O in Fig. 1). In addition, the coordinates of various other associated points (points A in Fig. 1) are also determined. Both approaches are employed, but the second approach is preferred where applicable.

Following are descriptions and illustrations of several different types of shorelines, descriptions of how different types of shorelines are defined, and descriptions of the geometric relationships between shorelines and median lines. Also included are descriptions and illustrations of the most commonly encountered shoreline/median line conditions that affect the shape and location of the median line which must be recognized and identified for computational purposes. In addition, analytical procedures and mathematical equations are presented that can be used to compute the coordinates of all points needed to determine the location of a median line and to prepare an accurate and complete legal description of a median line.

TYPES OF SHORELINES

Two different types of median line/shoreline relationships are considered; (1) median lines associated with shorelines defined by isolated shoreline points or clusters of shoreline points only, and (2) median lines associated with shorelines defined by both shoreline meander traverses and by isolated points or clusters of points. In the case of shorelines defined by points only, the median line consists of a series of intersecting straight line segments only. When shorelines are defined by meander traverses only, or by meander traverses and by points, the median line consists of a series of intersecting straight line segments and/or curved line segments.

SHORELINES DEFINED BY SALIENT POINTS

If a shoreline is defined only by the coordinates of prominently located points (such as large rocks or boulders scattered along or near the water's edge of a very irregular shoreline, for example, the shoreline is described as

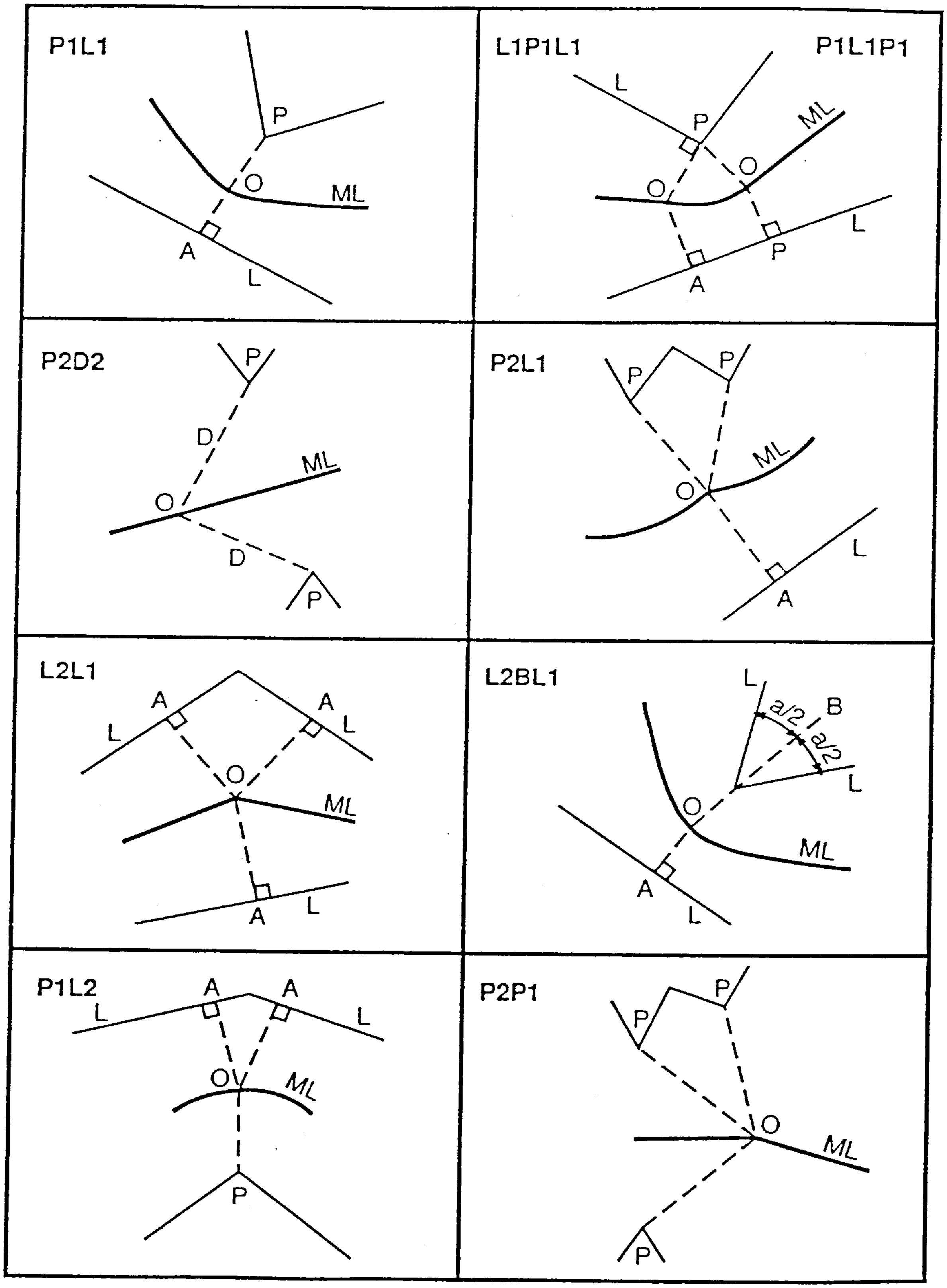


FIGURE 1. Most frequently encountered shoreline conditions with median line angle points and associated points, plus corresponding computational-option abreviations.

being defined by salient points. In such cases the median line consists of a series of intersecting straight line segments only. A shoreline of this type and the associated median line are shown in Fig. 2B.

Straight line segments of the median line, such as segments RS and ST in Fig. 2B, represent a portion of the median line every point of which is equidistant from two salient points only (point 110 and point 213 in the case of segment RS), one point on each opposite shoreline.

Median line direction changes occur abruptly as angle points at the points of intersection of adjoining straight line segments as occurs at point S in Figure 2B at the point of intersection of segments RS and ST.

The location of this type of median line can be defined accurately and completely in terms of the coordinates of median line angle points and end points only.

SHORELINES DEFINED BY MEANDER LINES AND BY SALIENT POINTS

If the shorelines are defined by meander lines only or by a combination of meander lines and salient points, the median line will consist of a series of intersecting straight line segments and/or curved line segments. A shoreline defined by meander lines only and the associated median line exists within the channel shown in Fig. 2A between median line points D and K. A shoreline defined by a combination of meander lines and salient points and the associated median line exists within the channel shown in Fig. 2A between median line points K and M.

Median line direction changes occur abruptly as angle points at the point of intersection of two straight line segments as occurs at point J in Fig. 2A, at the point of intersection of a nontangent curved line segment and a straight line segment as occurs at point D, and at the point of intersection of two nontangent curved line segments as occurs at point L. Direction changes can also occur gradually along curved line segments such as along curved line segment EFGHI, at the point of intersection of a straight line segment that is tangent to a curved line segment as occurs at point E, or at the point of intersection of two tangent curved line segments.

Straight median line segments exist where a median line passes between two straight shoreline meander line segments, i.e., where all points of the median line are equidistant from two straight shoreline meander line segments on opposite shorelines (such as along segment IJ in Fig. 2A. Straight median line segments also exist where a median line passes between two opposite shoreline meander line angle points that are convex toward the median line, or where a median line passes between two salient points, such as along segment ABC in Fig. 2A.

Curved median line segments are usually parabolic; all points of which are equidistant from a shoreline salient point or from a shoreline meander line angle point which is convex toward the median line on one shoreline, and a straight meander line segment on the opposite shoreline. Segment EFGHI in Fig. 2A ia a typical curved median line segment.

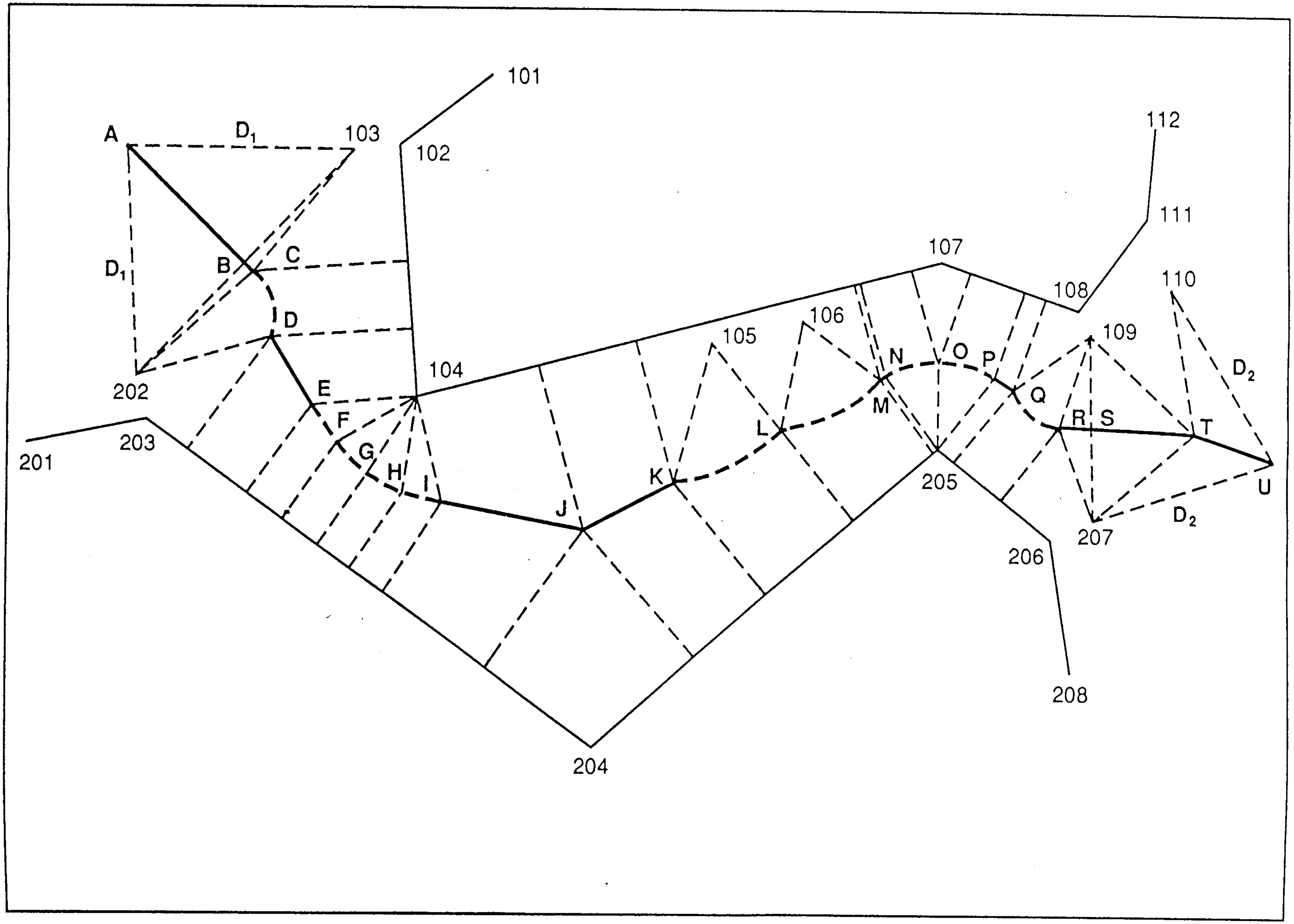


FIGURE 2A. Median line associated with a shoreline defined by meander lines and by salient points.

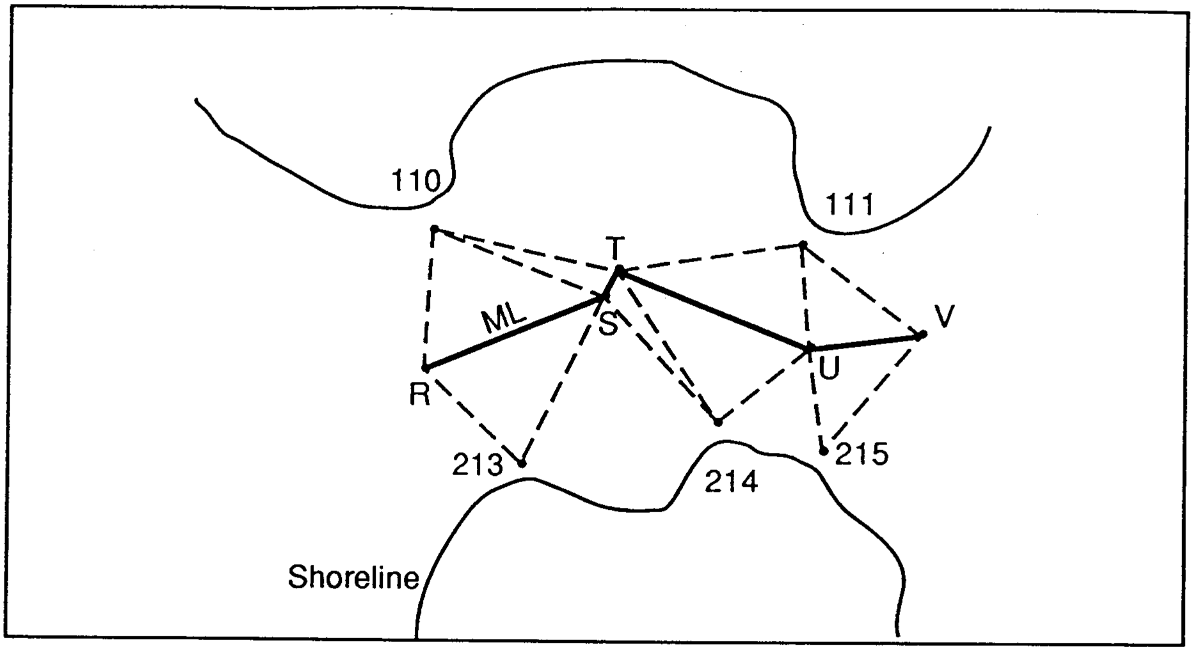


FIGURE 2B. Median line associated with a shoreline defined by salient points only.

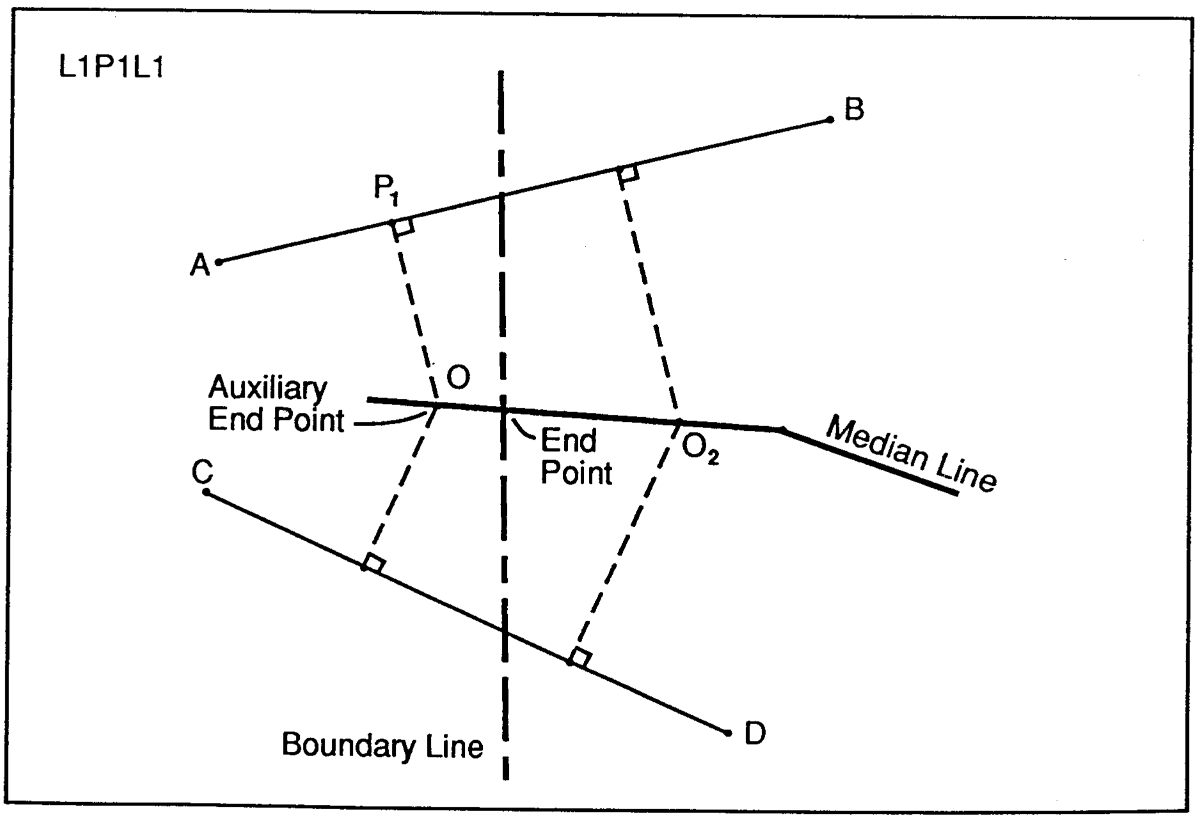


FIGURE 3. End point and auxiliary end point on straight line segment of median line associated with a shoreline defined by meander lines only.

Median line curves can be simple parabolic curves consisting of independent curved line segments bounded by straight line segments, such as segment EFGHI, or can be relatively complex curves consisting of two or more intersecting curved line segments such as segment NOP in Fig. 2A.

When the shoreline is defined by meander lines or by a combination of meander lines and salient points the location of the median line can be defined accurately in terms of the coordinates of the median line end points, the coordinates of median line angle points, and the coordinates of median line segment end points, supplemented by the coordinates of a small number of additional points-on-curves which are needed to define uniquely the shapes and locations of the curved median line segments.

MEDIAN LINE DEFINING POINTS AND ASSOCIATED POINTS

Median line defining points are points on a median line the coordinates of which are needed to accurately define the location of the median line. Defining points include median line beginning and ending points, straight line segment end points, curved line segment end points, points of intersection of adjoining curved and/or straight line segments, median line angle points, and points-on-curves.

Median line associated points are points on shoreline meander line segments or nondefining points on the median line, the coordinates of which are needed for computations associated with the determination of the coordinates of median line defining points or for computations needed to relate the location of the median line to the locations of nearby boundary lines, survey lines, shoreline meanders, etc.

MEDIAN LINE END POINTS

The actual points of beginning or ending of a median line are usually points of intersection between the median line and (1) an offshore boundary line in the case of an offshore median line, or (2) a land survey line in the case of an onshore median line. The coordinates of the actual end points are determined by computing the coordinates of the points of intersection of the median line and the limiting boundary lines.

In order to compute the coordinates of the actual end points it is sometimes helpful to compute the coordinates of one or more associated points on the median line. If these points are located near the probable location of the actual end points they can be used to isolate a localized segment of the median line in the vicinity of the boundary line which will assist in computing the coordinates of the actual end points. The nondefining end points of, and points on, localized median line segments are referred to as auxiliary end points.

<u>Shorelines Defined by Salient Points Only</u> [Option P2D2]. In the case of shorelines defined by salient points only, an auxiliary end point would be a point on a straight line segment of the median line which is (1) equidistant from two salient points that are located on opposite shorelines, and (2) which is also located a chosen distance from the two salient points. Point A in

Fig. 2A, for example, is an auxiliary end point that is located a distance D_1 from the two shoreline points 103 and 202. One or more points A might be computed in the vicinity of an offshore boundary line (by varying the distance D_1) to assist in computing the coordinates of the point of intersection of the median line with the offshore boundary line.

The coordinates of auxiliary end points, such as A can be determined utilizing the equations for point O presented in the Computational Option P2D2 section of the Appendix.

Also, in the case of an offshore boundary line, the actual median line endpoint can sometimes be determined in a similar manner if (1) the distance D_1 is equal to the breadth of the territorial sea, (2) the offshore boundary line at the point of intersection with the median line consists of circular arcs, and (3) the salient points which determine the location of the median line also determine the location of the offshore boundary at the point of intersection.

Shorelines Defined by Meander Lines Only, or By a Combination of Meander Lines and Salient Points [Options LIPILI and PILIPI]. In the case of shorelines defined by meander lines only and where the median line passes between two straight shoreline meander line segments on opposite shorelines, an auxiliary end point would lie on a straight line segment of the median line and would be equidistant from (1) the straight shoreline meander line segment on one shoreline, and (2) a selected point having known coordinates which lies on the straight shoreline meander line segment on the opposite shoreline. Point O in Fig. 3, for example, is an auxiliary end point that is equidistant from shoreline meander line segment CD on one shoreline and from point P_1 on shoreline meander line segment AB on the opposite shoreline.

One or more points O might be computed in the vicinity of a boundary line (by varying the location of Point P_1) to assist in computing the coordinates of the point of intersection of the median line with the boundary line. The coordinates of auxiliary end points such as O can be determined utilizing the equations for point O presented in the computational option LIPILI section of the Appendix.

In the case of a median line that passes between a straight shoreline meander line segment on one shoreline and either a salient point or a shoreline meander line angle point that is convex toward the median line on the opposite shoreline the median line would be curved.

When the median line endpoint is the point of intersection of a curved line segment of the median line with a boundary line, three or more auxiliary end points on the median line curve in the vicinity of the boundary line would provide sufficient information to define the location of a localized segment of the median line curve. This would assist in computing the coordinates of the point of intersection of the median line with the boundary line.

In this case an auxiliary end point would lie on the curved line segment of the median line and would be equidistant from (1) a selected point on the straight shoreline meander line segment on one shoreline, and (2) the salient point or convex angle point on the opposite shoreline. Point O in Fig. 4, for example, is an auxiliary end point that is equidistant from point P_2 on the straight shoreline meander line segment CD on one shoreline, and from point P_1 on the opposite shoreline.

Three or more points O computed in the vicinity of a boundary line (by varying the location of Point P_2) would assist in computing the coordinates of the point of intersection of the median line with the boundary line.

The coordinates of auxiliary end points such as O can be determined utilizing the equations for point O presented in the computational Option PILIPI section of the Appendix.

To be sure that the end point of a median line is located accurately in relation to the shorelines, the shorelines should be extended a sufficient distance beyond the desired end of the median line. This is because the median line location can be influenced by shoreline points or line segments that might be located some distance away from the median line. Therefore, shoreline salient points should be selected, and/or shoreline meander traverses should be extended well beyond enclosing boundary lines.

MEDIAN LINE ANGLE POINTS

Median line angle points occur (1) at the point of intersection of adjoining straight median line segments, (2) at the point of intersection of adjoining curved median line segments with straight median line segments that are not tangent to one another, and (3) at the point of intersection of adjoining curved line segments that are not tangent to one another.

Shoreline Defined By Salient Points Only

Points of Intersection of Straight Line Segments [Option P2P1]. In the case of a shoreline defined by salient points only where all median line segments are straight line segments the median line angle points are points of intersection of adjoining straight median line segments and are equidistant from three salient points. One pair of salient points determines the location of one straight median line segment, a second pair of salient points determines the location of the intersecting straight median line segment, and one of each of the two pairs of salient points is the same point (i.e., one salient point is common to both pairs.) Angle Point T in Fig. 5, for example, is the point of intersection of median line segment ST (which is equidistant from salient points P_2 and P_3) and median line segment TU (which is equidistant from salient points P_1 and P_3). In this case point P_3 is the common salient point, and angle point T is equidistant from all three salient points P_1, P_2, and P_3. The coordinates of angle points such as T can be determined utilizing the equations for point O presented in the Computational Option P2P1 section of the Appendix.

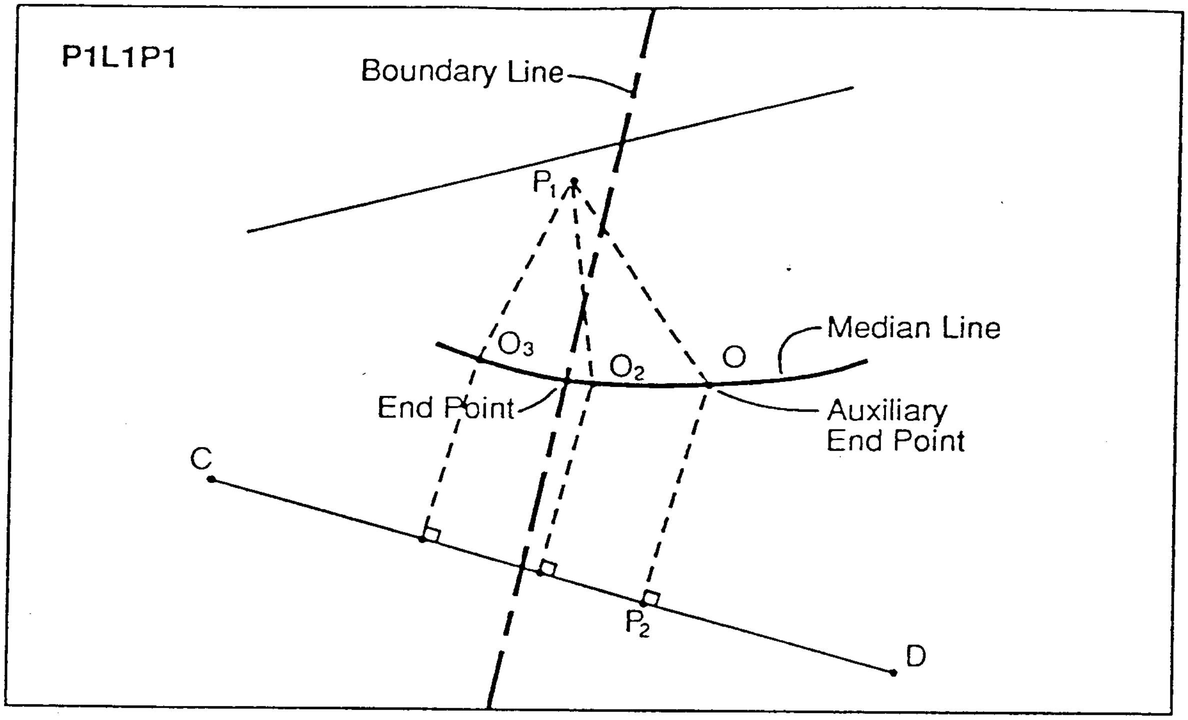


FIGURE 4. End point and auxiliary end points on curved line segment of a median line associated with a shoreline defined by meander lines and by salient points.

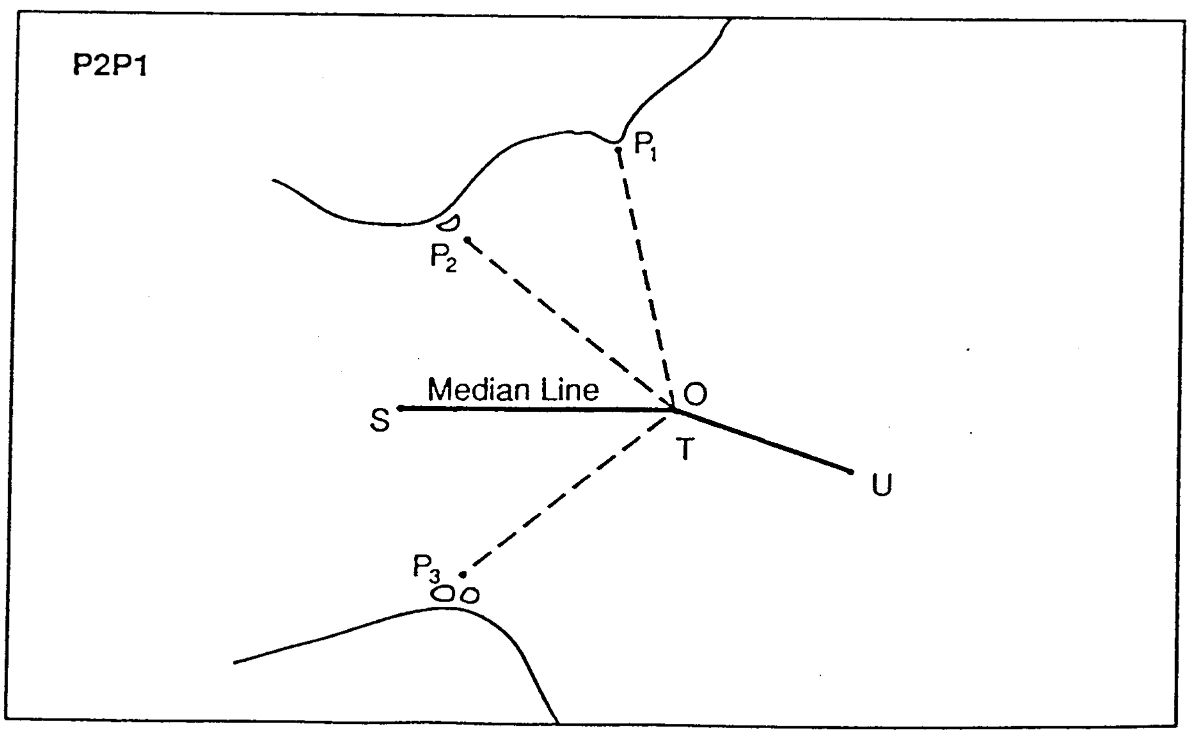


FIGURE 5. Median line angle point associated with a shoreline defined by salient points only.

Shoreline Defined By Meander Lines Only

Points of Intersection of Straight Line Segments[Option L2L1]. In the case of shorelines defined by meander lines only; straight line segments of the median line exist where the location of the median line is determined by two straight shoreline meander line segments, one on each opposite shoreline. Median line direction changes occur at angle points when the median line passes between two straight shoreline meander line segments connected by a meander line angle point which is concave toward the median line on one shoreline and one straight shoreline meander line segment on the opposite shoreline. At a median line angle point of this type the angle point is equidistant from all three of the straight shoreline meander line segments. Angle point J in Fig. 6, for example, is the point of intersection of median line segment IJ (which is equidistant from shoreline meander line segments EF and AB) with median line segment JK (which is equidistant from shoreline meander line segments EF and CD). In this case angle point J is equidistant from all three shoreline meander line segments AB, CD, and EF. (Shoreline meander line points B and C might or might not be the same point.)

The coordinates of angle points such as J can be determined utilizing the equations for point O presented in the Computational Option L2L1 section of the Appendix.

Points of Intersection of a Straight Line Segment and a Curved Line Segment [Option L1P1L1]. - In the case of shorelines defined by meander lines only, curved line segments of the median line exist where the location of the median line is determined by a shoreline meander line angle point that is convex toward the median line on one shoreline and by a straight shoreline meander line segment on the opposite shoreline. Straight line segments of the median line exist where the location of the median line is determined by two straight shoreline meander line segments, one on each opposite shoreline. Median line direction changes occur gradually along a parabolic curve when a straight line segment of the median line intersects a curved line segment of the median line at a point of tangency. This occurs when the location of both a curved line segment of the median line and a straight line segment of the median line are determined by a common straight shoreline meander line segment. In this case, the point of intersection (i.e., point of tangency) is equidistant from the common straight shoreline meander line segment, the straight shoreline meander line segment on the opposite shoreline, and the angle point. Point F in Fig. 7, for example, is the point of intersection of the straight median line segment EF (which is equidistant from straight shoreline meander line segments AB and CD) with the curved median line segment FG (which is equidistant from shoreline meander line point P_1 and from the common straight shoreline meander line segment CD). (Shoreline meander line point B and angle point P_1 might or might not be the same point).

The coordinates of points of intersection such as F can be determined utilizing the equations for point O presented in the Computational Option L1P1L1 section of the Appendix.

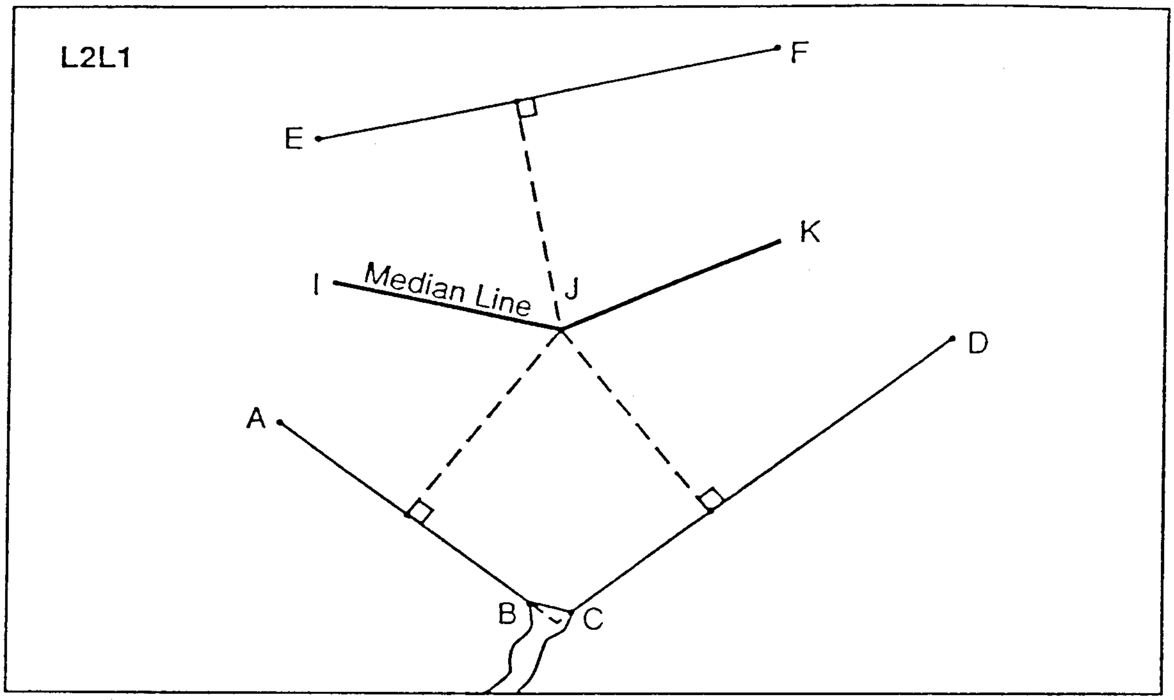


FIGURE 6. Median line angle point associated with a shoreline defined by meander lines only.

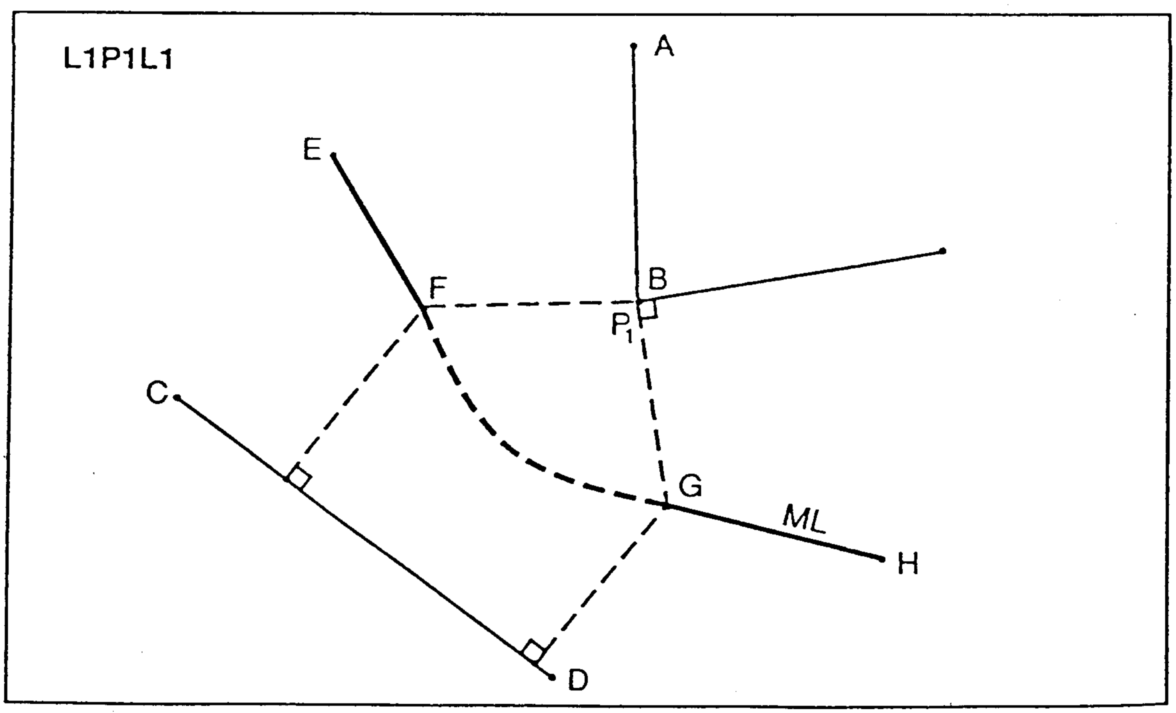


FIGURE 7. Point of intersection of a curved median line segment with a straight median line segment at a point of tangency associated with a shoreline defined by meander lines only.

Points of Intersection of Curved Line Segments[Option P1L2]. - In the case of shorelines defined by meander lines only, curved line segments of the median line exist where the location of the median line is determined by a meander line angle point that is convex toward the median line on one shoreline and by a straight shoreline meander line segment on the opposite shoreline. Median line direction changes occur abruptly when curved median line segments intersect adjoining curved median line segments that are not tangent to one another. This occurs when the location of two adjoining curved line segments of the median line are determined by a common meander line angle point that is convex toward the median line on one shoreline and by two different straight shoreline meander line segments connected by a meander line angle point which is concave toward the median line on the opposite shoreline. The common convex angle point and the two straight shoreline meander line segments determine the locations of the two adjoining curved median line segments. At a median line angle point of this type, the angle point is equidistant from the common convex shoreline meander line angle point and from both of the two straight shoreline meander line segments. Angle point O in Fig. 8, for example, is the point of intersection of curved median line segment NO (which is equidistant from shoreline meander line angle point P_3 and from straight shoreline meander line segment AB) with curved median line segment OP (which is equidistant from the common convex shoreline meander line angle point P_3 and from straight shoreline meander line segment CD). In this case angle point O is equidistant from both straight shoreline meander line segments AB and CD and from the common shoreline angle point P3. (Shoreline meander line points B and C might or might not be the same point.)

The coordinates of angle points such as point O can be determined utilizing the equations for point O presented in the Computational Option PIL2 section of the Appendix.

Shoreline Defined By Meander Lines and By Salient Points

Points of Intersection of a Straight Median Line Segment and a Curved Median Line Segment[Option P1L2 and P2L1] - In the case of shorelines defined by meander lines and by salient points, curved line segments of the median line exist where the location of the median line is determined by a shoreline salient point on one shoreline and by a straight shoreline meander line segment on the opposite shoreline. Straight line segments of the median line exist where the location of the median line is determined by two straight shoreline meander line segments, one on each opposite shoreline. Median line direction changes occur abruptly when a straight line segment of the median line intersects a curved line segment of the median line (at a point that might not be a point of tangency.) In this case the point of intersection is equidistant from the salient point and from both of the straight shoreline meander line segments. Angle Point G in Fig. 9, for example, is the point of intersection of curved median line segment FG (which is equidistant from shoreline salient point P_3 and from straight shoreline meander line segment CD) with straight median line segment GH (which is equidistant from straight shoreline meander line segments AB and CD). In this case angle point G is equidistant from both straight shoreline meander line segments AB and CD and from shoreline salient point P3.

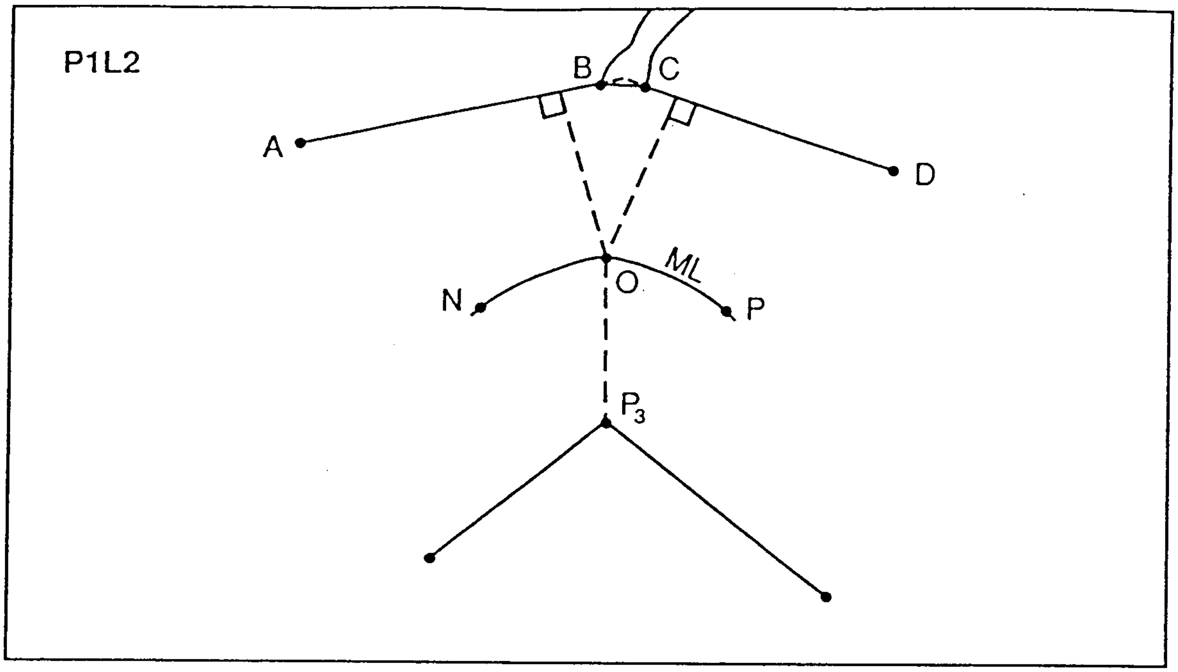


FIGURE 8. Point of intersection of nontangent curved median line segments associated with a shoreline defined by meander lines only.

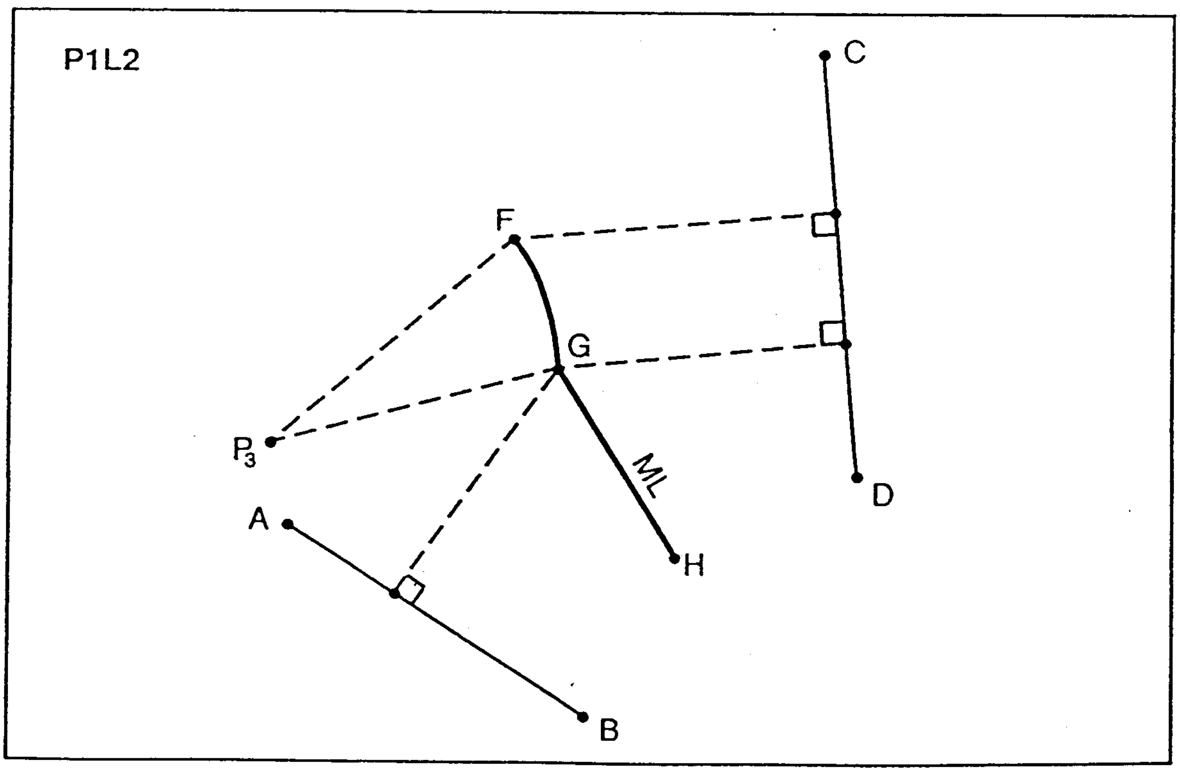


FIGURE 9. Point of intersection of a curved median line segment and a straight median line segment (which might not be tangent) associated with a shoreline defined by meander lines and by salient points.

The coordinates of angle points such as G can be determined utilizing the equations for point O presented in the Computational Option P1L2 section of the Appendix.

Also, in the case of shorelines defined by meander lines and by salient points, straight line segments of the median line will exist wherever the median line passes between two salient points (i.e., where the location of the median line is determined by two salient points that are located on opposite shorelines.) In this case the point of intersection of the straight line segment and the curved line segment is equidistant from the two salient points and from the straight shoreline meander line segment which determines the location of the curved line segment of the median line. Angle point Q in Fig. 10, for example, is the point of intersection of straight median line segment PQ (which is equidistant from salient points P_2 and P_3) with curved median line segment QR (which is equidistant from salient point P_3 and from straight shoreline meander line segment AB). In this case point Q is equidistant from the two salient points P_2 and P_3 and from the straight shoreline meander line segment AB.

The coordinates of angle points such as Q can be determined utilizing the equations for point O presented in the Computational Option P2LI section of the Appendix.

<u>Point of Intersection of Curved Line Segments</u> [Option P2L1] - In the case of shorelines defined by meander lines and by salient points, curved line segments of the median line exist where the location of the median line is determined by a salient point on one shoreline, and by a straight shoreline meander line segment on the opposite shoreline. Median line direction changes occur abruptly when curved median line segments intersect adjoining curved median line segments that are not tangent to one another. This occurs when the location of two adjoining curved line segments of the median line are determined by: (1) a common straight shoreline meander line segment on one shoreline; and (2) by two different shoreline salient points on the opposite shoreline. The straight shoreline meander line segment and one salient point determine the location of one of the curved median line segments; and the same straight shoreline meander line segment and the other salient point determine the location of the second curved median line segment. At a median line angle point of this type the angle point is equidistant from the common straight shoreline meander line segment and from both of the two shoreline salient points. Angle point L in Fig. 11, for example, is the point of intersection of curved median line segment KL (which is equidistant from shoreline salient point P_2 and from straight shoreline meander line segment AB) with curved median line segment LM (which is equidistant from shoreline salient point P_3 and from the common straight shoreline meander line segment AB.) In this case angle point L is equidistant from both salient points P_2 and P_3 and from the common straight shoreline meander line segment AB.

The coordinates of angle points such as L can be determined utilizing the equations for point O presented in the Computational Option P2L1 section of the Appendix.

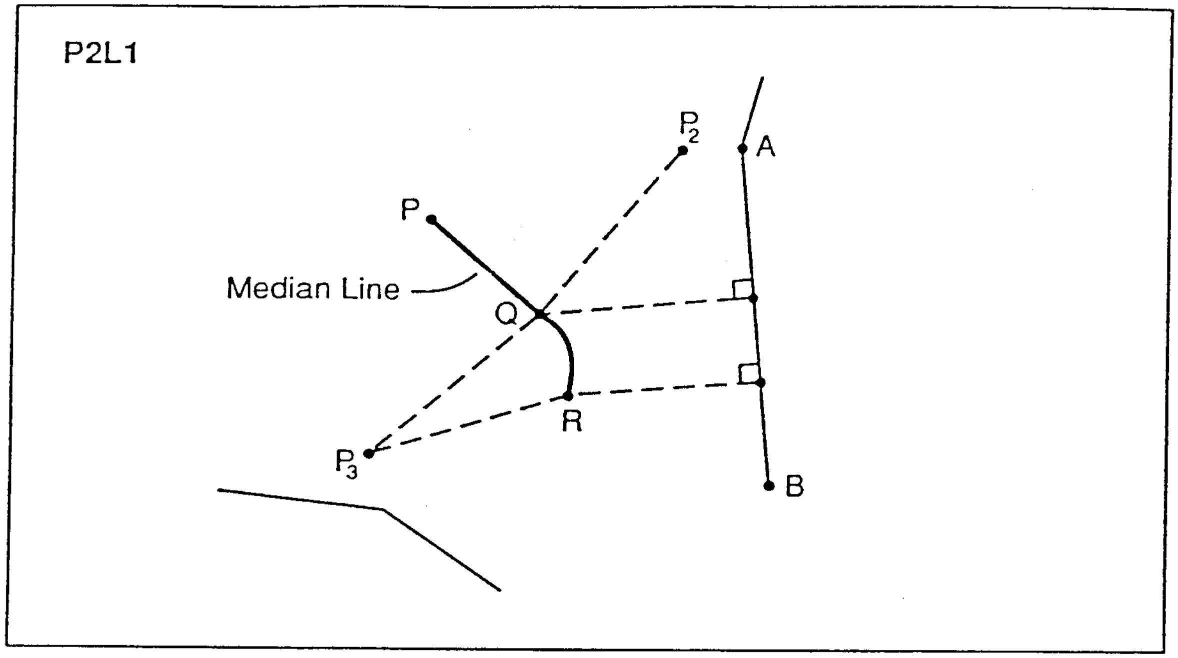


FIGURE 10. Point of intersection of a curved median line segment and a straight median line segment (which might not be tangent) associated with a shoreline defined by meander lines and by salient points.

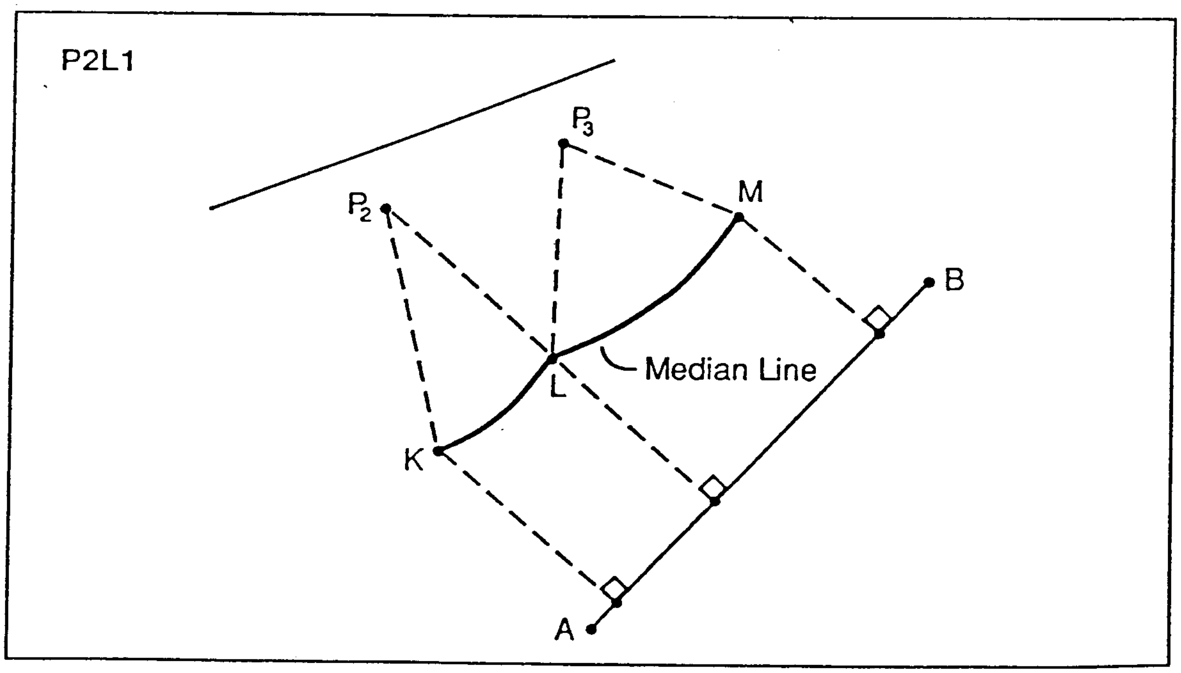


FIGURE 11. Point of intersection of nontangent curved median line segments associated with a shoreline defined by meander lines and by salient points.

POINTS-ON-CURVES [Options P1L1, L2BL1, and P1L1P1]

In order to accurately define the shape and location of a curved line segment of a median line the coordinates of one or more points on the curved line segment of the median line are needed - in addition to the coordinates of the end points. Since a median line curve is equidistant from a shoreline salient point or from a convex shoreline meander line angle point on one shoreline and from a straight shoreline meander line segment on the opposite shoreline, the angle point or salient point plus several other selected points located on the straight shoreline meander line segment are utilized to determine the approximate locations of points on the curve the coordinates of which can be computed accurately and conveniently.

One convenient choice for the location of a point-on-curve includes the point on the curve which lies on the line that is perpendicular to the straight shoreline meander line segment on one shoreline, and which also passes through the salient point or the convex shoreline meander line angle point on the opposite shoreline. Point D in Fig. 12 is a point-on-curve of this type.

The coordinates of points-on-curves such as point D can be determined using the equations for point O presented in the Computational Option P1L1 section of the Appendix.

Another convenient choice is the point on the curve which lies on the bisector of the convex shoreline meander line angle point. Point C in Fig. 12 is a point-on-curve of this type.

The coordinates of points-on-curves such as point C can be determined using the equations for point O presented in the Computational Option L2BL1 section of the Appendix.

Other choices might correspond to points on the straight shoreline meander line segment that are at the midpoint or quarter points of the portion of the shoreline meander line segment that lies between those points which correspond to the end points of the median line curved line segment. Points B and E in Fig. 12 are points-on-curve that correspond to the quarter points B1 and E1 of the straight shoreline meander line segment which lies between shoreline meander line points A1 and F1. A1 and F1 correspond to the end points A and F of the curved line segment of the median line. (Points A1 through F1 in Fig. 12 are points on the straight shoreline meander line segment from which perpendiculars pass through corresponding points-on-curve A through F.

The coordinates of points-on-curves such as B and E can be determined using the equations for point O presented in the Computational Option P1L1P1 section of the Appendix. The coordinates of curve end points such as A and F can be determined using the equations presented in the appropriate Computational Option section of the Appendix. (For example, in the case of a straight line/curve point of tangency as shown in Fig. 12 the coordinates of end points such as points A and F can be computed using the equations for point O presented in the Computational Option L1P1L1 section of the Appendix.)

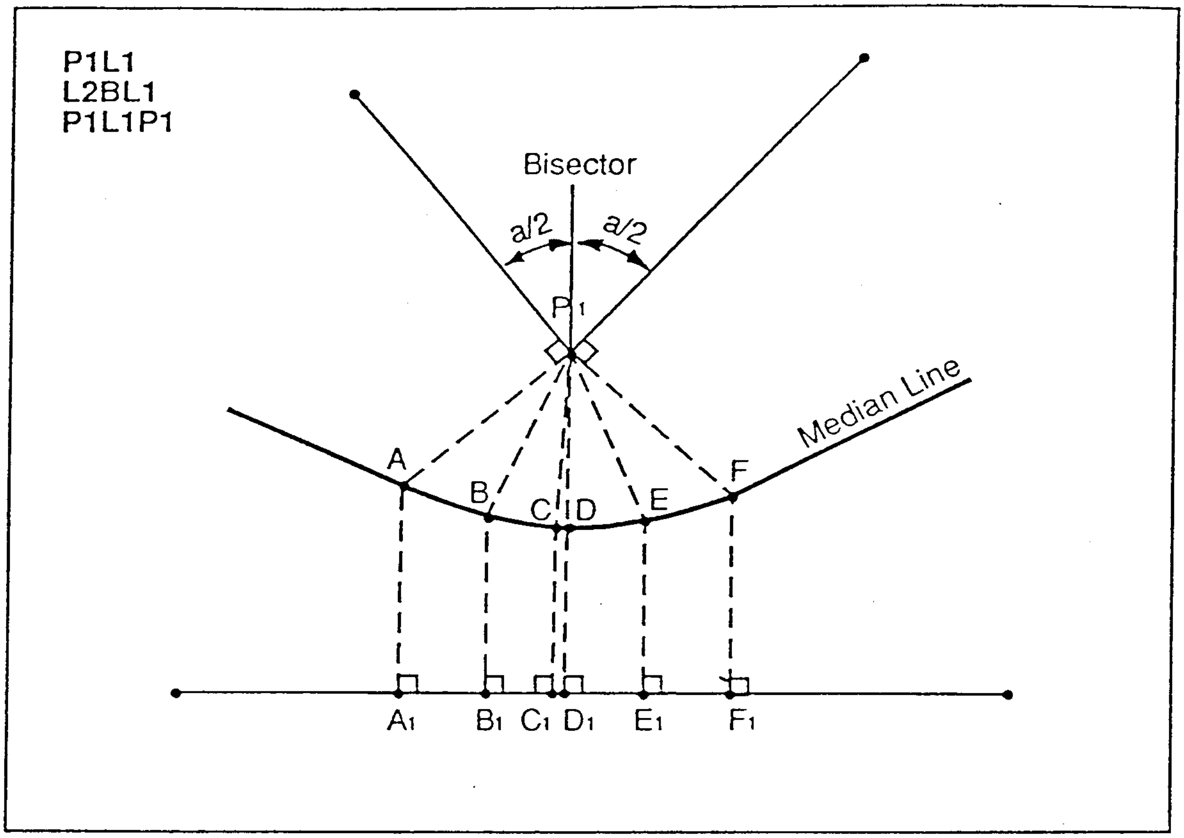


FIGURE 12. Defining points on a curved median line segment and associated points on the straight shoreline meander line segment.

IDENTIFYING THE CORRECT COMPUTATIONAL OPTION AND CORRECT MULTIPLE SOLUTION

To be sure that the correct coordinate computational equations are utilized, the geometric shoreline/median line computational option must be identified correctly. (In the preceding paragraph, for example, it was mentioned that the coordinates of the particular type of curve end points associated with a point of tangency such as points A and F should be computed using the Computational Option L1P1L1 equations.)

Similar appearing shoreline/median line conditions might be represented mathematically by significantly different equations. For example, the curve end points F in Fig. 7 and Q in Fig. 10 might appear to be the same (i.e. a straight line/curve) type of point of intersection; but the two are actually different geometrically, and the coordinates of the two end points are computed using different sets of equations (L1P1L1 and P2L1).

Also, different appearing shoreline/median line conditions might be represented mathematically by the same equations. For example, the coordinates of the points of intersection O in Fig 8 and G in Fig. 9 which appear to be different are both computed using the same set of equations (P1L2). Therefore, in determining the computational option to apply the underlying geometric shoreline/median line relationship must be recognized.

To assist in matching shoreline/median line conditions with corresponding computational options the shoreline/median line conditions that have been described and illustrated separately are summarized graphically in Fig. 1, and the computational-option abbreviations accompany the first line of each description, the description illustration, and the corresponding Fig. 1 diagram.

In addition, valid multiple solutions are obtained for the coordinates of many computed points. In some cases the desired pair of coordinates can be identified relatively easily; but in other cases the preferred solution is not obvious, and some additional logic is required to make the proper selection.

Because of the necessity for selecting the correct computational option (based occasionally upon subtle differences) and for identifying the correct solution (which might sometimes require considerable judgement) it is difficult to fully automate scientific computations of this type safely. For this reason, the existing computer programs have been designed to depend upon human interaction.

SUMMARY

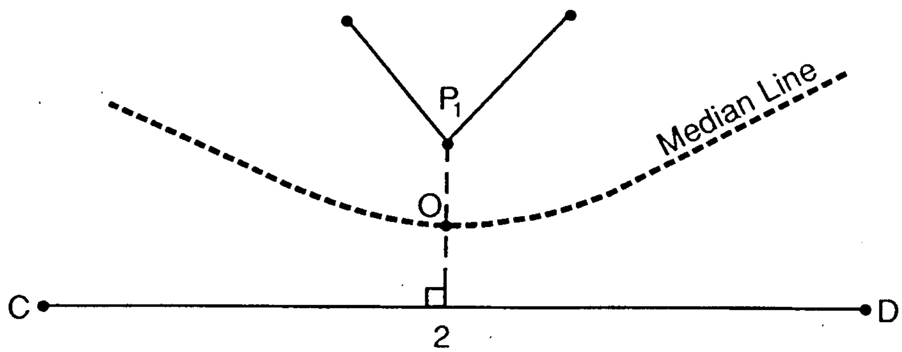
A geometric median line, lateral line, or equidistant line consists of a series of intersecting curved line segments and/or straight line segments every point of which is equidistant from the closest point on opposite shorelines.

Shorelines are sometimes defined by meander traverses and are sometimes defined by prominently located salient points only, or by combinations of meander traverses and salient points. The shape and location of an equidistant line is determined by the shape and location of the shoreline, and the type of equidistant line is determined by the type of shoreline definition.

A semianalytical process is utilized for computing the locations of equidistant lines in which shoreline plots are examined, shoreline/equidistant line conditions and corresponding computational options are identified, and the coordinates of all points needed to determine the accurate location of and to prepare a legal description of an equidistant line are determined.

Descriptions and illustrations of the most commonly encountered shoreline/equidistant line conditions plus the corresponding mathematical equations that are used to compute the coordinates of all needed equidistant line points are presented.

COMPUTATIONAL OPTION P1L1



Given: The x, y coordinates of point C (x_c, y_c), point D (x_d, y_d), and point $P_1(x_1, y_1)$.

Compute: The x, y coordinates of point 2 (x_2, y_2) and point 0 (x_0, y_0).

$$y_2 = \frac{(X_1 - B_1)(x_d - x_c) + y_1(y_d - y_c)}{M_1(x_d - x_c) + (y_d - y_c)}, \quad \text{and}$$

$$x_2 = M_1 y_2 + B_1. \quad \text{Also,}$$

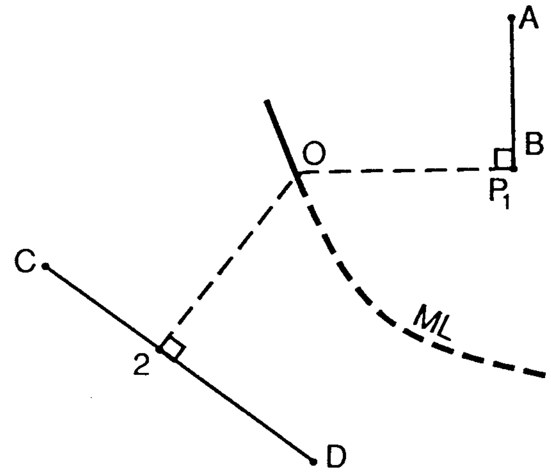
$$x_0 = 1/2(x_1 + x_2), \quad \text{and}$$

$$y_0 = 1/2(y_1 + y_2); \quad \text{in which}$$

$$M_1 = \frac{x_d - x_c}{y_d - y_c} \qquad y_d \neq y_c, \quad \text{and}$$

$$B_1 = x_c - M_1 y_c$$

COMPUTATIONAL OPTION L1P1L1



Given: The x, y coordinates of point $A(x_a, y_a)$, point $B(x_b, y_b)$, point $P_1(x_1, y_1)$, point $C(x_c, y_c)$, and point $D(x_d, y_d)$.

Compute: The x, y coordinates of point $2(x_2, y_2)$ and point $O(x_0, y_0)$.

$$y_2 = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}, \quad \text{and}$$

$$x_2 = M_2 y_2 + B_2 . \quad \text{Also}$$

$$y_0 = G y_2 + H , \quad \text{and}$$

$$x_0 = M_0 y_0 + B_0 ; \quad \text{in which}$$

$$M_0 = \frac{y_a - y_b}{x_b - x_a} \qquad x_b \neq x_a$$

$$B_0 = x_1 - M_0 y_1$$

$$M_2 = \frac{x_d - x_c}{y_d - y_c} \qquad y_d \neq y_c$$

$$B_2 = x_c - M_2 y_c$$

$$F = M_0(x_c - x_d) + (y_c - y_d)$$

$$G = \frac{M_2(x_c - x_d) + (y_c - y_d)}{F}$$

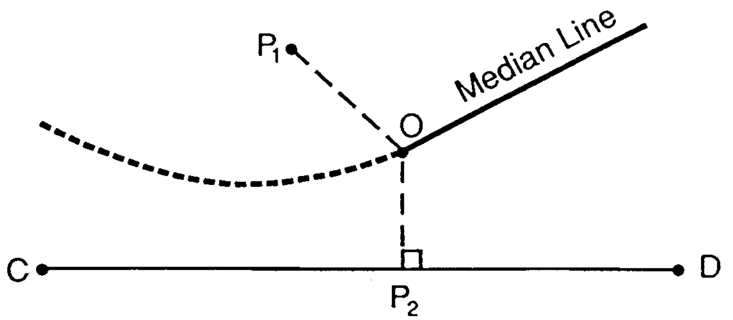
$$H = \frac{(B_2 - B_0)\,(x_c - x_d)}{F}$$

$$A = 2G(M_0M_2 + 1) - (M_2^2 + 1)$$

$$B = 2\,\{G[M_0(B_2 - x_1) - y_1] + H[M_0M_2 + 1] + M_2[B_0 - B_2]\}\text{ , and}$$

$$C = 2M_0H(B_2 - x_1) + B_2(2B_0 - B_2) + x_1(x_1 - 2B_0) + y_1(y_1 - 2H)\ .$$

COMPUTATIONAL OPTION P1L1P1



Given: The x, y coordinates of point P_1 (x_1, y_1), point C (x_c, y_c), point D (x_d, y_d), and point P_2 (x_2, y_2).

Compute: The coordinates of point 0 (x_0, y_0).

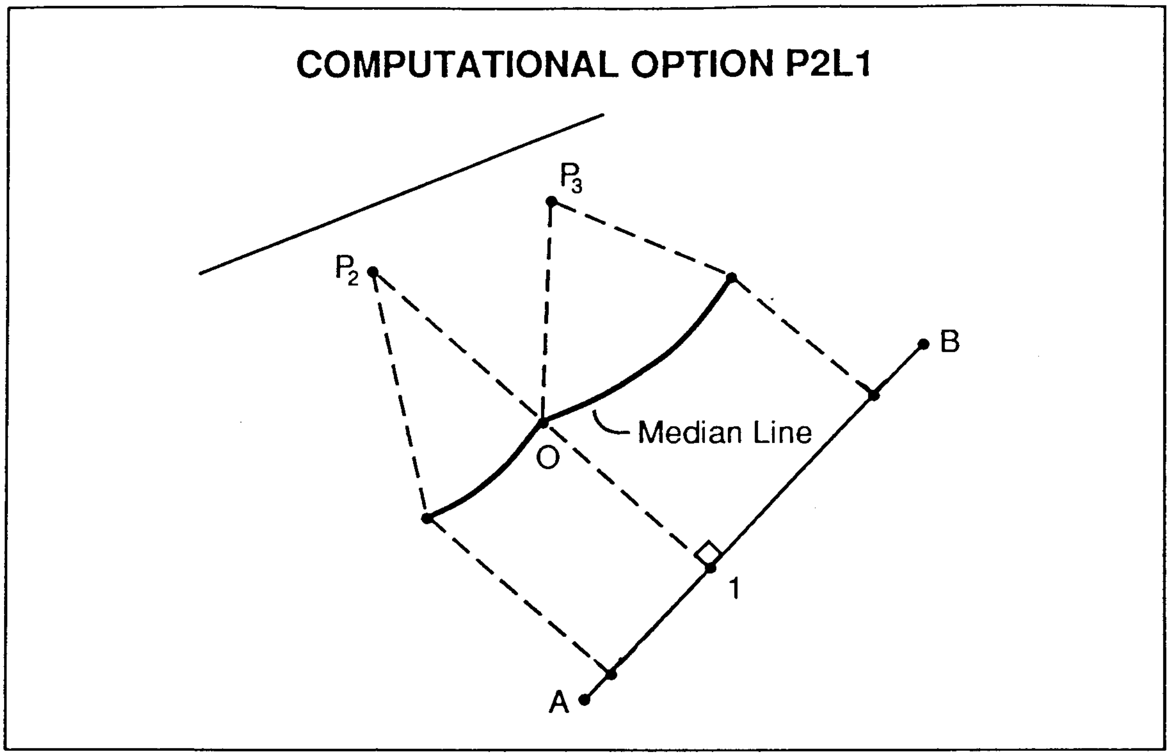
$$y_0 = \frac{2B_2(x_1 - x_2) - H}{2[M_2(x_2 - x_1) + (y_2 - y_1)]}, \quad \text{and}$$

$$x_0 = M_2 y_0 + B_2; \quad \text{in which}$$

$$M_2 = \frac{y_a - y_b}{x_b - x_a} \quad x_b \neq x_a$$

$$B_2 = x_1 + M_1 y_1 \quad \text{, and}$$

$$H = x_1^2 + y_1^2 - x_2^2 - y_2^2 .$$



Given: The x, y coordinates of point A (x_a, y_a), point B (x_b, y_b), point P_2 (x_2, y_2), and point P_3 (x_3, y_3).

Compute: The x, y coordinates of point 0 (x_0, y_0) and point 1 (x_1, y_1).

$$y_0 = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}, \quad \text{and}$$

$$x_0 = M_1 y_0 + B_1. \quad \text{Also,}$$

$$y_1 = M_3 y_0 + B_3, \quad \text{and}$$

$$x_1 = M_2 y_1 + B_2; \quad \text{in which}$$

$$M_1 = \frac{y_2 - y_3}{x_3 - x_2} \qquad x_2 \neq x_3$$

$$B_1 = \frac{x_3^2 + y_3^2 - x_2^2 - y_2^2}{2(x_3 - x_2)}$$

$$M_2 = \frac{x_b - x_a}{y_b - y_a} \qquad y_b \neq y_a$$

$$B_2 = x_a - M_2 y_a$$

$$M_3 = \frac{M_1 M_2 + 1}{M_2^2 + 1}$$

$$B_3 = \frac{M_2(B_1 - B_2)}{M_2^2 + 1}$$

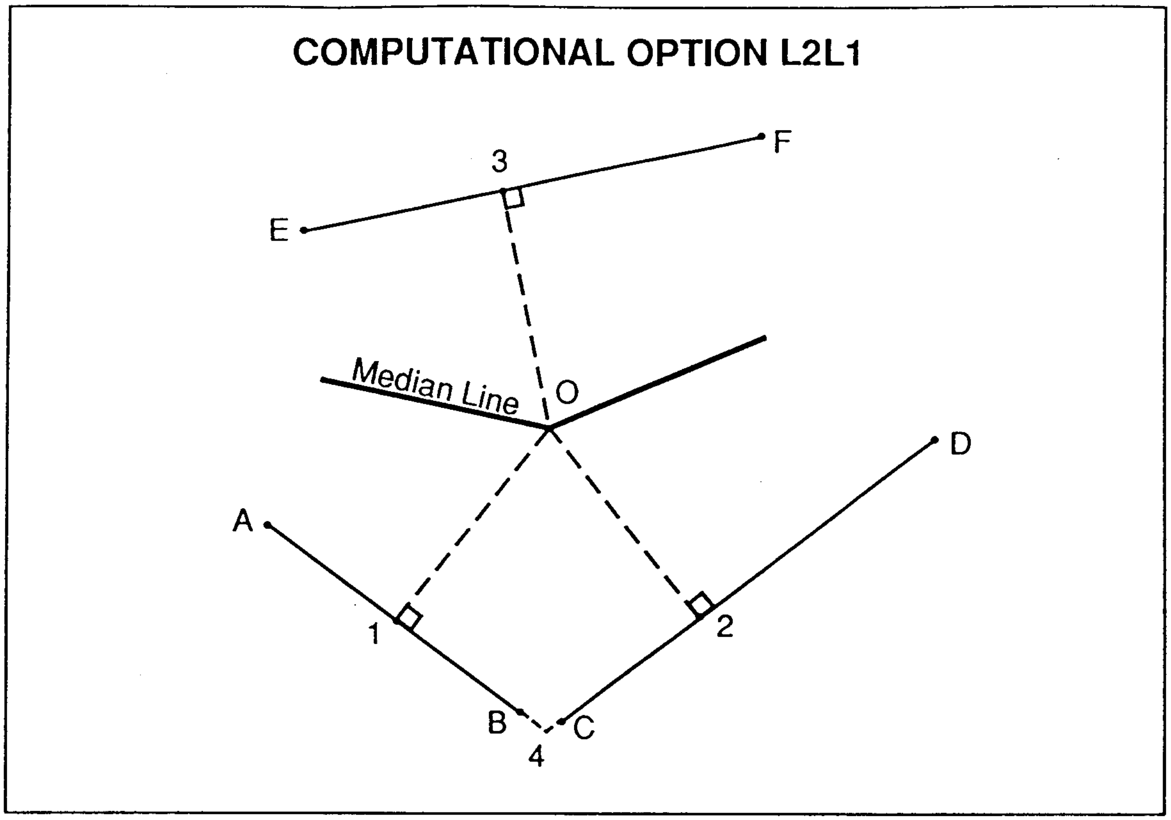
$$M_4 = M_2 M_3$$

$$B_4 = M_2 B_3 + B_2$$

$$A = M_4^2 + M_3^2 - 2(M_1 M_4 + M_3)$$

$$B = 2[M_1(x_3 - B_4) + M_4(B_4 - B_1) + B_3(M_3 - 1) + y_3] \text{ , and}$$

$$C = B_3^2 - x_3^2 - y_3^2 + B_4^2 + 2B_1(x_3 - B_4) \text{ .}$$



Given: The x,y, coordinates of point A (x_a, y_a), point B (x_b, y_b), point C (x_c, y_c), point D (x_d, y_d), point E (x_e, y_e) and point F (x_f, y_f).
(Points B, C, and 4 can be the same point.)

Compute: The x,y coordinates of point 1 (x_1, y_1), point 2 (x_2, y_2), point 3 (x_3, y_3), and point 0 (x_o, y_o).

$$y_o = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}, \quad \text{and}$$

$$x_o = M_o y_o + B_o . \qquad \text{Also}$$

$$y_1 = T y_o + U$$

$$x_1 = M_1 y_1 + B_1$$

$$y_2 = P y_o + Q$$

$$x_2 = M_2 y_2 + B_2$$

$$y_3 = R y_o + S \ , \qquad \text{and}$$

$$x_3 = M_3 y_3 + B_3 \ ; \qquad \text{in which}$$

$$M_1 = \frac{x_b - x_a}{y_b - y_a} \qquad y_b \neq y_a$$

$$M_2 = \frac{x_d - x_c}{y_d - y_c} \qquad y_d \neq y_c$$

$$M_3 = \frac{x_e - x_f}{y_e - y_f} \qquad y_e \neq y_f$$

$$B_1 = x_a - M_1 y_a$$

$$B_2 = x_c - M_2 y_c$$

$$B_3 = x_f - M_3 y_f$$

$$y_4 = \frac{M_1 y_a - M_2 y_d + x_d - x_a}{M_1 - M_2} \qquad M_1 \neq M_2$$

$$x_4 = M_1(y_4 - y_a) + x_a$$

$$D_a = [(x_a - x_4)^2 + (y_a - y_4)^2]^{1/2}$$

$$D_d = [(x_d - x_4)^2 + (y_d - y_4)^2]^{1/2}$$

$$x_g = \frac{D_d}{D_a}(x_a - x_4) + x_4$$

$$y_g = \frac{D_d}{D_a}(y_a - y_4) + y_4$$

$$M_o = \frac{y_g - y_d}{x_d - x_g} \qquad x_d \neq x_g$$

$$B_o = x_4 - M_o y_4$$

$$G = \frac{1}{(M_1^2 + 1)}$$

$$H = M_1 G$$

$$I = -HB_1$$

$$J = \frac{1}{(M_2^2 + 1)}$$

$$K = M_2 J$$

$$L = -KB_2$$

$$M = \frac{1}{M_3^2 + 1}$$

$$N = M_3M$$

$$O = -NB_3$$

$$P = J + KM_0$$

$$Q = KB_0 + L$$

$$R = M + NM_0$$

$$S = NB_0 + O$$

$$T = G + HM_0$$

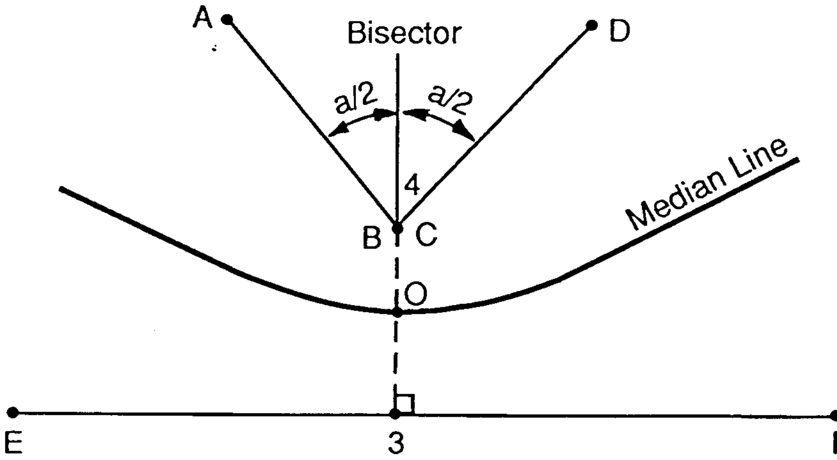
$$U = HB_0 + I$$

$$A = 2M_0(M_2P - M_3R) + R^2(M_3^2 + 1) - P^2(M_2^2 + 1) + 2(P-R)$$

$$B = 2[M_0(M_2Q + B_2 - M_3S - B_3) + B_0(M_2P - M_3R) + RS(M_3^2 + 1) - PQ(M_2^2 + 1) + Q - S + B_3M_3R - B_2M_2P] \quad \text{, and}$$

$$C = 2[B_0(M_2Q + B_2 - M_3S - B_3) + B_3M_3S - B_2M_2Q] + S^2(M_3^2 + 1) - Q^2(M_2^2 + 1) + B_3^2 - B_2^2.$$

COMPUTATIONAL OPTION L2BL1



Given: The x, y coordinares of point $A(x_a, y_a)$, point $B(x_b, y_b)$, point $C(x_c, y_c)$, point $D(x_d, y_d)$, point $E(x_e, y_e)$, and point $F(x_f, y_f)$.
(Points B, C, and 4 can be the same point.)

Compute: The x, y coordinates of point $O(x_0, y_0)$ and point $3(x_3, y_3)$.

$$y_0 = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}, \quad \text{and}$$

$$x_0 = M_0 y_0 + B_0. \quad \text{Also}$$

$$y_3 = M_5 y_0 + B_5, \quad \text{and}$$

$$x_3 = M_3 y_3 + B_3 \ ; \quad \text{in which}$$

$$M_1 = \frac{x_b - x_a}{y_b - y_a} \qquad y_b \neq y_a$$

$$M_2 = \frac{x_c - x_d}{y_c - y_d} \qquad y_c \neq y_d$$

$$M_3 = \frac{x_f - x_e}{y_f - y_e} \qquad y_f \neq y_e$$

$$B_3 = x_e - M_3 y_e$$

$$y_4 = \frac{M_1 y_a - M_2 y_d + x_d - x_a}{M_1 - M_2}$$

$$x_4 = M_1(y_4 - y_a) + x_a$$

$$D_a = [(x_a - x_4)^2 + (y_a - y_4)^2]^{1/2}$$

$$D_d = [(x_d - x_4)^2 + (y_d - y_4)^2]^{1/2}$$

$$x_g = \frac{D_d}{D_a}(x_a - x_4) + x_4$$

$$y_g = \frac{D_d}{D_a}(y_a - y_4) + y_4$$

$$x_5 = 1/2\,(x_g + x_d)$$

$$y_5 = 1/2\,(y_g + y_d)$$

$$M_0 = \frac{x_5 - x_4}{y_5 - y_4} \qquad y_5 \neq y_4$$

$$B_0 = x_4 - M_0 y_4$$

$$M_5 = \frac{M_0(x_f - x_e) + (y_f - y_e)}{M_3(x_f - x_e) + (y_f - y_e)}$$

$$B_5 = \frac{(B_0 - B_3)(x_f - x_e)}{M_3(x_f - x_e) + (y_f - y_e)}$$

$$M_6 = M_3 M_5$$

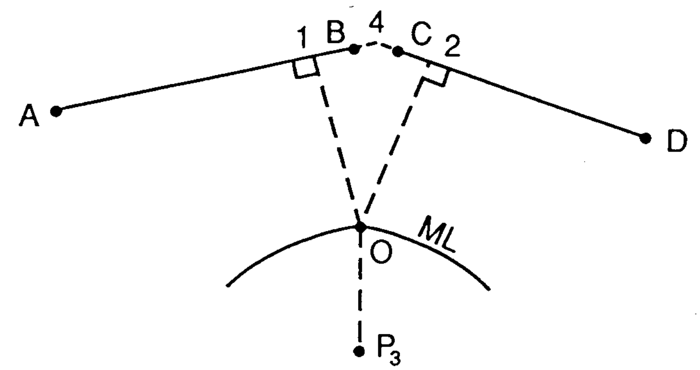
$$B_6 = M_3 B_5 + B_3$$

$$A = M_6(M_6 - 2M_0) + M_5(M_5 - 2)$$

$$B = 2[M_0(x_4 - B_6) + M_6(B_6 - B_0) + B_5(M_5 - 1) + y_4]\ , \quad \text{and}$$

$$C = 2B_0(x_4 - B_6) + B_5^2 + B_6^2 - x_4^2 - y_4^2\,.$$

COMPUTATIONAL OPTION P1L2



Given: The x, y coordinates of point $A(x_a, y_a)$, point $B(x_b, y_b)$, point $C(x_c, y_c)$, point $D(x_d, y_d)$, and point $P_3(x_3, y_3)$.
(Points B, C, and 4 can be the same point.)

Compute: The x, y coordinates of point $O(x_0, y_0)$, point $1(x_1, y_1)$, and point $2(x_2, y_2)$.

$$y_0 = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}, \quad \text{and}$$

$$x_0 = M_0 y_0 + B_0. \quad \text{Also}$$

$$y_1 = Ty_0 + U$$

$$x_1 = M_1 y_1 + B_1$$

$$y_2 = Py_0 + Q, \quad \text{and}$$

$$x_2 = M_2 y_2 + B_2; \quad \text{in which}$$

$$M_1 = \frac{x_b - x_a}{y_b - y_a} \quad y_b \neq y_a$$

$$M_2 = \frac{x_d - x_c}{y_d - y_c} \quad y_d \neq y_c$$

$$B_1 = x_a - M_1 y_a$$

$$B_2 = x_c - M_2 y_c$$

$$y_4 = \frac{M_1 y_a - M_2 y_d + x_d - x_a}{M_1 - M_2}$$

$$x_4 = M_1(y_4 - y_a) + x_a$$

$$D_a = [(x_a - x_4)^2 + (y_a - y_4)^2]^{1/2}$$

$$D_d = [(x_d - x_4)^2 + (y_d - y_4)^2]^{1/2}$$

$$x_g = \frac{D_d}{D_a}(x_a - x_4) + x_4$$

$$y_g = \frac{D_d}{D_a}(y_a - y_4) + y_4$$

$$M_0 = \frac{y_g - y_d}{x_d - x_g} \qquad x_d \neq x_g$$

$$B_0 = x_4 - M_0 y_4$$

$$G = \frac{1}{M_1^2 + 1}$$

$$H = M_1 G$$

$$I = -HB_1$$

$$J = \frac{1}{M_2^2 + 1}$$

$$K = M_2 J$$

$$L = -KB_2$$

$$P = J + KM_0$$

$$Q = KB_0 + L$$

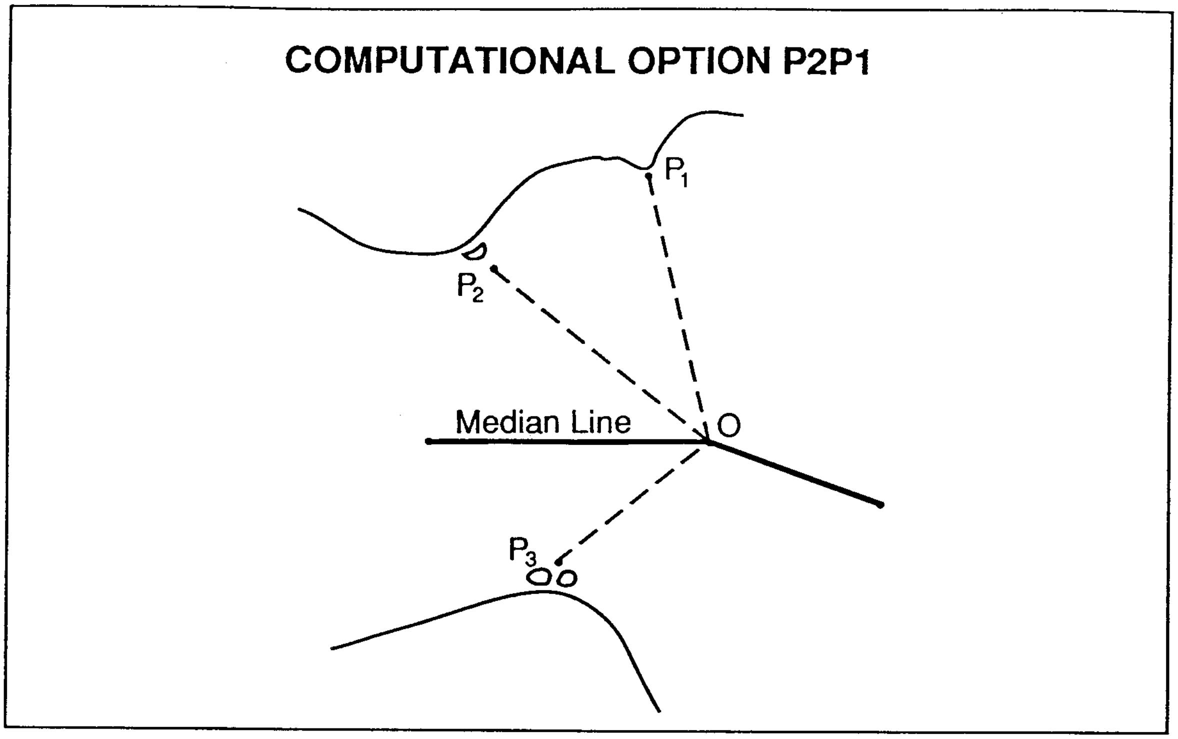
$$T = G + HM_0$$

$$U = HB_0 + I$$

$$A = 2P(M_0 M_2 + 1) - P^2(M_2^2 + 1)$$

$$B = 2[M_0(M_2 Q + B_2 - x_3) + P(M_2 B_0 - M_2 B_2 - M_2^2 Q - Q) + Q - y_3], \text{ and}$$

$$C = 2B_0(M_2 Q + B_2 - x_3) - B_2(2M_2 Q + B_2) - Q^2(M_2^2 + 1) + x_3^2 + y_3^2 .$$



Given: The x, y coordinates of point $P_1(x_1, y_1)$, point $P_2(x_2, y_2)$, and point $P_3(x_3, y_3)$.

Compute: The coordinates of point $O(x_0, y_0)$.

$$y_0 = \frac{G(x_2 - x_1) - F(x_3 - x_1)}{2E}, \quad \text{and}$$

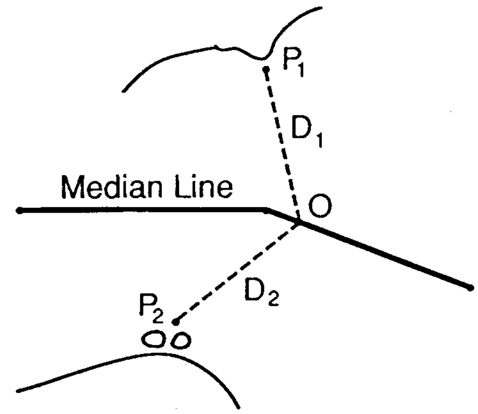
$$x_0 = \frac{F - 2y_0(y_2 - y_1)}{2(x_2 - x_1)} \quad x_2 \neq x_1\text{; in which}$$

$$E = (y_3 - y_1)(x_2 - x_1) - (y_2 - y_1)(x_3 - x_1)$$

$$F = x_2^2 + y_2^2 - x_1^2 - y_1^2, \quad \text{and}$$

$$G = x_3^2 + y_3^2 - x_1^2 - y_1^2 .$$

COMPUTATIONAL OPTION P2D2



Given: The x, y coordinates of point $P_1(x_1, y_1)$, point $P_2(x_2, y_2)$, and distances D_1 and D_2 (D_1 and D_2 can be equal).

Compute: The coordinates of point $O(x_0, y_0)$.

$$y_0 = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}, \qquad \text{and}$$

$$x_0 = M_0 y_0 + B_0 \; ; \qquad \text{in which}$$

$$M_0 = \frac{y_2 - y_1}{x_1 - x_2} \qquad x_1 \neq x_2$$

$$D = D_2^2 - D_1^2 + y_1^2 - y_2^2 + x_1^2 - x_2^2$$

$$B_0 = \frac{D}{2(x_1 - x_2)}$$

$$A = M_0^2 + 1$$

$$B = 2(B_0 M_0 - y_1 - M_0 x_1), \qquad \text{and}$$

$$C = x_1^2 + y_1^2 + B_0^2 - 2B_0 x_1 - D_1^2 .$$