Implementation of Lagrangian Stochastic Models to Parameterize Submesoscale Transport for Tracking Oil Spills in the Gulf of Mexico
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Prepared under BOEM Contract
M11PC00034
by
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ABOUT THE COVER
Example of particle-based tracer (in orange) in the presence of submesoscale flows in the central Gulf of Mexico. The tracer was launched in the vicinity of the Deepwater Horizon site and was advected by HYbrid Coordinate Ocean Model surface velocities at a one kilometer resolution. The attracting Lagrangian coherent structures of the model are plotted in dark blue.

ACKNOWLEDGEMENTS
We greatly acknowledge the support of the Bureau of Ocean and Energy Management under Contract No. M11PC00034 for sponsoring this study. We thank in particular Walter Johnson for the very insightful comments on the Gulf of Mexico circulation, and our Contract Officer Representative Zhen Li. We would also like to thank Pat Hogan and Greg Jacobs, our colleagues at the Naval Research Laboratory for the high-resolution simulations they provided.

This research also was made possible, in part, by a grant from British Petroleum/The Gulf of Mexico Research Initiative (GoMRI) and by the Office of Naval Research. We thank the Consortium for Advanced Research on Transport of Hydrocarbon in the Environment (CARTHE) team for their contribution to the Grand LAgrangian Deployment (GLAD) drifter data set and analysis, which is part of the CARTHE program, supported by GoMRI.
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Abbreviations and Acronyms

AVISO Archiving, Validation and Interpretation of Satellite Oceanographic (data)
CARTHE Consortium for Advanced Research on Transport of Hydrocarbon in the Environment
DWH Deepwater Horizon
FSLE Finite Scale Lyapunov Exponent
GLAD Grand LAgrangian Deployment
GoM Gulf of Mexico
HYCOM HYbrid Coordinate Ocean Model
LAF Louisiana-Alabama-Florida shelf
LCS Lagrangian Coherent Structures
LSM Lagrangian stochastic models
LTS Louisiana-Texas shelf
NCOM Navy Coastal Ocean Model
NRL Naval Research Laboratory
OGCM Ocean General Circulation Model
psd power spectrum density
Rd Radius of deformation
SCULP Surface Current Lagrangian drift Program
SMS Submesoscales
std standard deviation
WES Western Gulf of Mexico
WFS West Florida shelf
1 Introduction

Submesoscale (SMS) flows refer to the types of motions in the spatial scale ranging from 100 m to about 10 km and time scales of few days only. They were first pointed out by McWilliams (1985) and have been recently the focus of intensive research. They are confined to the mixed-layer and can emerge from instabilities developing on the edges of fronts (Özgökmen et al., 2011, 2012). In the presence of mesoscale features, the controlling factor for baroclinic mixed-layer instabilities in the Gulf Stream was found to be primarily the depth of the mixed layer (Mensa et al., 2013). Resulting SMS features tend to emerge in winter and fall, when the weak surface stratification and deep mixed-layer allow for vertical recirculations along the flanks of fronts. Similarly, there is increasing evidence that the surface circulation in the Gulf of Mexico (hereafter GoM) is influenced by SMS motions, and that they have an impact on particle distributions (Zhong et al., 2012).

The present work focuses on the statistical relative dispersion as a metric quantifying the impact of multi-scale motions on the surface dispersion of particles. The two-particle metrics are more connected to turbulent and scalar fluctuations in the underlying flow field. The time-dependent relative dispersion $D^2(t)$, is defined as:

$$D^2(t) = \langle (r_2(t) - r_1(t))^2 \rangle ,$$  \hspace{1cm} (1)

and measures the averaged squared distance between two particles.

Another relative dispersion metric is the scale-dependent Finite Scale Lyapunov Exponent (or FSLE) $\lambda(\delta)$, introduced by Artale et al. (1997) and Aurell et al. (1997) as:

$$\lambda(\delta) = \frac{log(\alpha)}{<\tau(\delta)>} ,$$  \hspace{1cm} (2)

where $1 < \alpha < 2$, and $<\tau(\delta)>$ is the averaged time taken by all particle-pairs to separate from the distances $\delta$ to $\alpha\delta$. Unlike $D^2(t)$, the FSLE separates the effect of each scale of motion on the relative dispersion, and is closely linked to the turbulent kinetic energy spectrum.

For surface ocean flows, the scale-dependent FSLE can be represented by the schematic diagram displayed in Fig. 1.1: at the mesoscales, the relative dispersion is local, meaning that it is governed by ocean features of the same scales, and follows a power-law usually between Richardson ($\sim \delta^{-2/3}$) and ballistic ($\sim \delta^{-1}$).
Figure 1.1: Diagram depicting the different types of regimes evidenced by the scale-dependent Finite Scale Lyapunov Exponent for surface ocean flows: $\lambda$ (units in days$^{-1}$) versus the spatial scale $\delta$. $\delta_S$ is a small-scale O(m), radius of deformation ($R_d$) is the first baroclinic radius of deformation, L is a large synoptic scale such as the oceanic basin scale. $\lambda_{I_{\text{max}}}$ and $\lambda_{II_{\text{max}}}$ correspond to the Lyapunov Exponents of a Non-Local versus Local types of dispersion, respectively.

For larger scales, such as basin scales where particle pair velocities are no longer correlated, the diffusive regime ($\sim \delta^{-2}$) is observed. At the submesoscales, the dispersion regimes are still unknown to date, although they have far-reaching implications for ocean prediction problems: if the SMS regime is non-local (Hypothesis-I in Fig. 1.1), the relative dispersion at the small scales is entirely controlled by the mesoscale features, imposing an exponential rate of separation ($\lambda(\delta) = \text{constant}$). In this scenario, mesoscale-eddy permitting ocean models are sufficient to resolve dispersion and mixing problems. On the other hand, a local regime such as the one depicted by Hypothesis-II indicates that the dispersion is controlled by features at the SMS, and mesoscale-permitting ocean models cannot capture small scale dispersion.

Relative dispersion measurements from in-situ data are still sparse and limited, in that historical drifters do not provide unbiased information on the small-scale dispersion of particle-pairs. A recent effort was made by the Consortium for Advanced Research on Transport of Hydrocarbon in the Environment (CARTHE, cf www.carthe.org), which was funded by the Gulf of Mexico Research Initiative to measure unbiased estimates of the scale-dependent relative dispersion. The Grand LAgangian Deployment (GLAD) was designed as a multi-scale dispersion experiment and consisted of fractal launches of more than 300 drifters in the De Soto Canyon area of the GoM. Relative dispersion estimates confirmed the presence of SMS motions from all cluster experiments in the launch locations, which are discussed in this report.

In numerical models, the scale-dependent FSLE at the SMS was found to be largely underestimated (Poje et al., 2010). The Lyapunov Exponent of the model ($\lambda_{\text{max}}$) is linearly...
dependent and can be inferred from the averaged strain rate $\tilde{Q}^+$ obtained from the Okubo-Weiss criterion (Okubo, 1970; Weiss, 1991), an Eulerian quantity defined as:

$$Q = S^2 - \omega^2,$$

$$S^2 = S_n^2 + S_s^2, \quad S_n = \frac{\partial u}{\partial x} - \frac{\partial v}{\partial y}, \quad S_s = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y},$$

where $u$ and $v$ are the model horizontal velocities, $S_n$ and $S_s$ are the normal and shear components of the strain, respectively. By considering only the positive values where regions are dominated by strain and deformation, one can define:

$$\overline{Q}^+ = A^{-1} \int \sqrt{Q} dA, \quad Q > 0,$$

with $A$ being the area of positive $Q$. It was found that refining the model resolution leads to an increase in $\tilde{Q}^+$, and this increase results in an increase in $\lambda_{\text{max}}$. When the horizontal resolution is high enough to resolve features of the order of few kilometers, the SMS motions are seen to enhance the rate of particle-pair separation at the small scales up to the first baroclinic radius of deformation ($R_d$) (Poje et al., 2010; Haza et al., 2012). This in turn can affect the spreading and mixing of a tracer both at its early and later stages. It is therefore important to include the SMS component into a Lagrangian prediction problem when using mesoscale-only permitting ocean models.

To do so, parameterizations are required to enhance the unrealistically low strain levels at the SMS range. In light of the increasing complexity of the transport barriers in high-resolution ocean models, however, these parameterizations must be of statistical nature for computational efficiency. Recent work (Haza et al., 2012) has focused on the combination of Markovian stochastic particle models for the SMS range, and deterministic Lagrangian Coherent Structures (LCS) for the mesoscale range. The objective is to correct statistically the relative dispersion at the SMS, while preserving the mesoscale transport pathways.

Three Lagrangian stochastic models (hereafter LSM) have been considered for relative dispersion correction in the GoM. Those models are all based on Markovian assumptions and single-particle statistics. The LSM-1 (Random walk) and LSM-2 (Random flight) are the classic LSMs, and have often been used in the oceanographic community for Lagrangian problems involving ocean models, altimetry or high-frequency (HF) radars. These subgridscale models add a stochastic component to the velocity field in order to mimic the effect of subgridscale motions on particle trajectories. They have been shown to be limited in their impact on particle pair separations when diffusion is injected incrementally at the time-integration frequency, and with velocity fluctuations an order of magnitude smaller than the typical velocity scales (Ullman et al., 2006; Haza et al., 2012). They generate a local regime in the scale-dependent FSLE ($\delta^{-1.7}$ power law) and high $\lambda$ values at the low SMS range, yet underestimate the rate of pair separations in the high SMS range and at the intermediate scales (around the Rd). Further increasing the diffusion, however, results in smearing the mesoscale pathways which control the general direction of the flow. Their performances are nevertheless satisfactory when applied to
the surface circulation of the continental shelf, or low kinetic energy flows.

The LSM-1 for single particle motion is the uncorrelated random walk and a 0th order Markov model that is Markovian in the distance only. It consists of adding a random kick $dx$ at every integration time step $Dt$ as follows:

\[ dx = L_K \, dW , \] (5)

where the random kick is characterized by an amplitude $L_K$ and a normal distribution $dW$ of zero-mean and standard deviation (std) of unity. It generates velocity fluctuations with a std $\sigma = L_K/Dt$, which corresponds to a diffusivity $K = L^2_K/(2Dt)$. A reasonable parameter choice in the GoM would be $L_K = 1 \, km$ and for a $Dt = 2 \, hrs$, $\sigma \approx 14 \, cm/s$ (1 order of magnitude smaller than the Loop current velocities), and $K=139 \, m^2/s$.

For two-particle motion, a space-correlation scale $L_D$ needs to be introduced, and the random kick of the second particle becomes:

\[ dx_2 = AA \, dx_1 + (1 - AA) \, L_K \, dW , \] (6)

where $dx_1$ is the random kick of the first particle computed from the above equation, and the kick of the second particle is constrained by:

\[ AA = \exp \left( -\frac{\delta^2}{2 \, L_D^2} \right) . \] (7)

$AA$ is a parameter representing the effect of submesoscale eddies on particle pair separations and constrains the random kick of the second particle from differing too much from the random kick of the first particle when their relative distance $\delta$ is within $L_D$ ($AA=1$ if both particles are at the same location). Typically $2 \, km \leq L_D \leq 5 \, km$ is sufficient to reduce the excessive accelerations of the uncorrelated random walk at the small scales, while still increasing significantly the rate of pair separations.

For $N$-particle motion, a random kick can be expressed as:

\[ dx = \frac{1}{N} \sum_{i=1}^{N} AA_i dx_i + \left( 1 - \frac{1}{N} \sum_{i=1}^{N} AA_i \right) \, L_K \, dW . \] (8)

The LSM-2 for single particle motion is the Random Flight (Griffa, 1996), Markovian in both the distance and velocity. It adds to the model velocity vector $v=(u,v)$ a random turbulent velocity vector $v'=(u',v')$, modulated in time by a decorrelation time scale $\tau$. The turbulent velocity increment along a particle trajectory in the zonal direction is given by:
where \( u' \) is the turbulent velocity at time \( t \), \( \sigma_u \) is the turbulent zonal velocity std and \( \tau_u \) is the decorrelation time-scale in the zonal direction.

The correspondence with LSM-1 in terms of diffusivity \( (K = \sigma_u^2 \tau_u \text{ for homogenous turbulence}) \) implies that, for a given decorrelation time scale, \( \tau_u \), \( \sigma_u \) should be chosen such that

\[
\sigma \sim \frac{L_K}{D t} \sqrt{\frac{D t}{2 \tau_u}},
\]

since the decorrelation time-scale of a random walk is \( D t/2 \) (Griffa, 1996).

The spatial correlation is also implemented by modifying the turbulent velocity increment of the second particle as follows:

\[
du'_2 = -u' \frac{D t}{\tau_u} + (1 - AA) \sigma_u \sqrt{\frac{2 D t}{\tau_u}} dW_1 + AA \sigma_u \sqrt{\frac{2 D t}{\tau_u}} dW,
\]

(11)

Where \( dW_1 \) is the random displacement of the first particle. Note that the spatial correlation does not have much impact on the relative dispersion of trajectories parameterized with LSM-2, since one can set the initial random displacement of the first particle to zero.

The LSM-3 was developed recently (Haza et al., 2007) and is also based on the Markov 1 approximation, but aims instead at modifying the Lagrangian statistical parameters of the ocean model. It requires therefore a different scale-separation, where the turbulent component is not entirely parameterized by a stochastic component. After decomposing the flow intrinsically, the Lagrangian velocity of a particle advected by the model is:

\[
\frac{dX_m}{dt} = U_m(X_m, t) + u'_m(X_m, t),
\]

(12)

where \( U_m \) is the filtered model velocity corresponding to the slowly evolving mesoscale field, and \( u'_m \) is the remaining turbulent velocity field. The parameterized (corrected) trajectories are obtained from the LSM-3 via the addition of the missing turbulent velocity component \( \eta(t) \), so that the corrected Lagrangian velocity becomes:

\[
\frac{dX_c}{dt} = U_m(X_c, t) + u'_m(X_c, t) + \eta(t).
\]

(13)

By assuming that some of the rotating component is already included in the resolved mesoscale eddies, the LSM-3 formulation for the two velocity components can be decoupled, and the time-dependence of the zonal component of \( \eta \) along a trajectory is given by the following expression:
\[
\frac{d\eta(t)}{dt} = a \frac{du_m'(t, r_c(t))}{dt} + bu_m'(t, r_c(t)) + c\eta(t),
\]

where

\[
a = \frac{\sigma_r \sqrt{\tau_m}}{\sigma_m \sqrt{\tau_r}} - 1, \quad b = \frac{\sigma_r}{\sigma_m \sqrt{\tau_r \tau_m}} - \frac{1}{\tau_r}, \quad c = -\frac{1}{\tau_r},
\]

with \((\sigma_r, \tau_r)\) and \((\sigma_m, \tau_m)\) being the realistic/true and model velocity fluctuations and Lagrangian correlation times, respectively. The variance of the missing component \(\eta\) is given by

\[
\eta^2 = \frac{(\sigma_r \sqrt{\tau_m} - \sigma_m \sqrt{\tau_r})^2 + (\sigma_r \sqrt{\tau_r} - \sigma_m \sqrt{\tau_m})^2}{\tau_r + \tau_m},
\]

which is zero if the model and the real parameters coincide. Provided an adequate flow-decomposition based on low-pass filters retaining most of the mesoscale velocities, the LSM-3 generates a local regime by enhancing the model’s Lyapunov Exponent by a factor of 2 to 3 at the most at the SMS. It can also enhance the FSLE at the intermediate scales to more realistic levels, without affecting the mesoscale pathways. This results in a substantial increase of the absolute and relative dispersions to levels higher than the LSM-1 and 2, while injecting correlated diffusion selectively on the edges of the mesoscale features (Haza et al., 2012).

As a reference for the LSM performance, we rely on the existing multi-scale dispersion estimates obtained from observations in the GoM (the GLAD and the Surface Current Lagrangian drift Program [SCULP] data sets), and on a SMS-permitting ocean model. The latter is a 1 km resolution (1/100\(^\circ\)) simulation with the HYbrid Coordinate Ocean Model (HYCOM) configured for the GoM, and run at the Naval Research Laboratory (NRL). It successfully resolves SMS features in the 3 to 10 km range, with indication of a pronounced seasonality. Although the Lyapunov exponent of this simulation is still below the GLAD estimates at the small scales, the intermediate-scale FSLEs of GLAD and HYCOM 1/100\(^\circ\) can be considered the same, which are an essential component of dispersion correction. For the continental shelf, FSLE estimates from the SCULP data-set are limited in that the experiment was not designed for SMS relative dispersion measurements, and thus lacks temporal resolution and drifter-pair availability at the SMS. We relied therefore on chance-pair computations with the knowledge that the relative dispersion estimates can be biased. Overall, we refer mainly to HYCOM 1/100\(^\circ\) throughout the LSM implementation, as it is more practical to evaluate the impact of the LSMs on the relative dispersion metrics in the entire GoM.

The mesoscale-permitting ocean model used in conjunction with the LSM is the NRL’s HYCOM 1/25\(^\circ\) free run. Both models have identical forcing conditions, which turn out to be an advantage for Lagrangian parameterization on the shelf.

The report is organized as follows: The relative dispersion from numerical models and recent observations (GLAD) is presented in Section 2. The implementation of the third Lagrangian stochastic model in the GoM interior is described in Section 3. This section also includes a LSM-3 application to the Archiving, Validation and Interpretation of Satellite Oceanographic data.
(AVISO) altimetric field with a comparison to the GLAD drifter trajectories. In Section 4, the three Lagrangian stochastic models are implemented on the continental shelf of the GoM. A summary and conclusion is presented in Section 5.
2 Dispersion regimes in HYCOM 1/25° and HYCOM 1/100°

2.1 Mesoscale dominated GoM interior

Since ocean general circulation models (OGCMs) can only resolve flow features as small as 2 to 3 times the grid-spacing, a 4-km resolution model has very limited capacity to reproduce circulation features below the Rd, which is about 30 to 50 km in the GoM interior, and 5 to 10 km on the shelf. As illustrated in Fig. 2.1, the surface salinity of a 1/25° HYCOM simulation shows mostly signatures of smooth mesoscale features, including the Loop Current system.

![Figure 2.1: Surface salinity snapshots from the HYCOM1/25° simulation around summer (left) and fall (right).](image)

On the other hand, the same model configured with a horizontal resolution of about 1 km (Fig. 2.2) can now resolve smaller eddies that are of the order of 10 km. Their presences are noticeable on the edges of the main features, such as the Loop current and south of the continental shelf near the Mississippi River outflow (top left and right panels, respectively). Although a 4 km resolution model can technically resolve 10 km features, these types of frontal instabilities occur at wavelengths below the grid-spacing. These instabilities are more prominent in winter and fall, when vertical circulations along the sides of fronts are allowed via a deep mixed-layer and weak stratification. In late spring and summer, the shallow mixed layer (Fig. 2.2 bottom panel), and the stronger upper-layer stratification prevents the formation of vertical recirculations, resulting in smoother fields such as the snapshot displayed in the middle panel of Fig. 2.2.

The SMS seasonality is illustrated also in both local and statistical FSLEs in Fig. 2.3 where one can see that the SMS features modify the attracting Lagrangian coherent structures (LCS) with smaller-scale patterns at day 44, while the smoother LCS of the mesoscale eddies dominate at day 170 in a manner very similar to coarser resolution models. The resulting scale-dependent FSLE ($\lambda(\delta)$) for particles launched every month confirms the SMS seasonality with maximum Lyapunov exponents at 0.6 day$^{-1}$ i.e., more than 1.5 times higher in winter than in summer with a
minimum at 0.35 day$^{-1}$. Note that the time-evolution of the positive partition of the Okubo-Weiss parameter (not shown) does not reflect any seasonality as is clearly the case with the Gulf Stream (Haza, 2012). It could be due to some interference by the Loop Current, since it is a prominent feature and likely to mask the seasonal variation in the horizontal stretching rate.

Figure 2.2 Salinity snapshots of the HYCOM1/100° run showing the development of instabilities on the edges of the mesoscale features around fall and winter (upper panel). In spring and summer, the shallower mixed-layer prevents the development of the SMS instabilities as illustrated in the middle panel. Bottom panel: Time evolution of the averaged mixed layer-depth given by HYCOM 1/100°.
Figure 2.3: Upper panel: backward FSLE maps in the GoM in winter (left) and late spring (right), illustrating the SMS seasonality. Units are in hour$^{-1}$. Lower panel: $\lambda(\delta)$ of two-month trajectories, color-coded by the launch day in 2010. Particles triplets were released every month over the GoM.

Comparisons of mesoscale and submesoscale permitting OGCMs, such as in this case or in the Gulf Stream system, point to the conclusion that while the mesoscale pathways still control the flow, the SMS features resulting from those instabilities affect the Lagrangian transport by ejecting particles from the edges of the mesoscale features (Haza et al., 2012). These particles then move along nearby mesoscale transport barriers and tend to “surf” those main pathways. This results in a substantial increase of the relative dispersion.
The scale-dependent FSLE of both models (Fig. 2.4) shows that the inclusion of the 2 to 10 km SMS increases the relative dispersion of particle-pairs when they are less than twice the first internal baroclinic Rd, as illustrated by the higher values of $\lambda(\delta)$ at the small scales. In winter (upper panel) when the SMS activity is expected to be the highest, the maximum Lyapunov exponent of the HYCOM1/100° run is seen to be twice the $\lambda_{\text{max}}$ of the HYCOM 1/25° run, confirming that the average hyperbolicity can be doubled when the grid-resolution is quadrupled. Yet their relative velocities are the same beyond those scales, due to the mesoscale control past twice the Rd. In summer (Fig. 2.4, lower panel), the difference in $\lambda_{\text{max}}$ is reduced due to the absence of mixed layer instabilities in HYCOM 1/100°. The discrepancy in the scale-dependent FSLE between the two models at the SMS and intermediate scales is the main focus of the LSM implementation in the GoM.

2.2 Dispersion regimes on the shelf

2.2.1 General model-observation comparison:

The HYCOM1/100° FSLE-map of Aug 20, 2010 (Fig. 2.5 top panel) illustrates the difference in the spatial patterns of the FSLE extrema between the continental shelf and the GoM interior, which act as transport barriers. This is an indication that the surface circulation on the shelf is not dominated by the mesoscale features of the GoM interior.

This was confirmed earlier by the results from LaCasce and Ohlmann (2003) from the SCULP drifter experiment, where releases of hundreds of drifters on the northern continental shelf yielded substantially different relative dispersion regimes, such as a weaker $\lambda_{\text{max}} \approx 0.3$ day$^{-1}$ and a transition to power-law at $\delta \approx 10$ km, that is more representative of the radius of deformation on the shelf.

The $\lambda(\delta)$ of the SCULP-II drifters is recomputed and compared to the FSLE obtained from particles launched on the Louisiana-Alabama and North Florida (hereafter LAF) shelf where the depth does not exceed 50 m (Fig. 2.5 bottom panel). To obtain a sufficiently high number of small-scale pairs available, $\lambda(\delta)$ was calculated from chance pairs (i.e., pairs forming by chance instead of being launched within a given distance) and the same was done with trajectories in the shelf and entire GoM of both HYCOM1/100° and 1/25° runs (the latter one is the assimilated version GOMI0.04/expt 30.1 in 2010 and 2011 (cf. [https://hycom.org/dataserver/goml0pt04/expt-30pt1](https://hycom.org/dataserver/goml0pt04/expt-30pt1)). Note that $\lambda$ values for SCULP-II are higher than the model values and reach 0.9-1 day$^{-1}$ for $\delta=1$ km. Those results confirm the tendencies of models to underestimate the relative dispersion at the small scales, in this case by a factor of 3 for the HYCOM1/25° at the 1km scale.
Figure 2.4: $\lambda(\delta)$ of both HYCOM1/25° and HYCOM1/100° free run trajectories in winter (Day 1) and summer (Day 181), computed from both original and chance pairs.
Figure 2.5: Top panel: Backward and forward FSLE map of the Northern GOM surface circulation in HYCOM1/100° at day 230 in 2010. Black areas are regions of weak dispersion or concentration. Bottom panel: $\lambda(\delta)$ of the full GoM and Louisiana-Alabama and North Florida (LAF) shelf from HYCOM1/25° (black line) and HYCOM1/100° (brown line) computed from chance pairs. Superimposed is the scale-dependent FSLE of the SCULP-II drifter data obtained from chance-pairs (342 surface drifters were deployed in 1996-1997 in the LAF shelf).

The model FSLE on the shelf also has a different dispersion regime than the full GoM FSLE, with a Richardson power-law ($\sim \delta^{-2/3}$) at $\delta > 10$ km similarly to SCULP-II when using chance pairs. This local regime indicates that the particle separations at these scales are governed by flow features of the same scales. $\lambda(\delta)$ also reaches the same maximum value of 0.2-0.3 day$^{-1}$ for
the full GoM, that is imposed by the average stretching rate. Note that if $\lambda$ is computed from original pairs on the shelf, it will result in much higher FSLE estimates at the intermediate scales beyond 20 km, sometimes reaching the full GoM values. The limitation of the advection duration fixed at 3 months allows only for the fastest pairs to reach the required ($\delta, a\delta$) distances. Most of the time, the fastest pairs are the particle-pairs entrained in the hyperbolic regions of the mesoscale eddies in the GoM interior.

Another significant difference between the in-situ drifters and model FSLE is that the SCULP-II FSLE is remarkably stable (not shown) throughout the duration of the experiment, while the model curve varies depending on the time of launch (not shown). $\lambda$ estimates at all scales are generally low compared to SCULP-II. Releases of particles on the shelf and advected with model velocities indicate that when the particles are pushed off-shore, the increase in kinetic energy and entrainment into the Loop Current eddies can result in an increase of $\lambda$ and a shift to the full GoM FSLE regime. In these instances, $\lambda(\delta)$ is closer to the SCULP-II FSLE. Conversely, particles pushed towards the coast result in the lowest FSLE estimates at $\delta > 10$ km. It is not clear at this point why the model FSLE for the LAF-shelf launch is more variable than the FSLE of the SCULP-II data. Unresolved missing physics in the model such as tides and mixed-layer issues could explain the model/data discrepancy. Despite these limitations, there is a clear transition from non-local (enstrophy/exponential) to local regimes occurring at scales around or below 10 km, which corresponds to the radius of deformation on the continental shelf (around 5-10 km).

### 2.2.2 Wind impact

In the following analysis of the shelf circulation, we focus on the influence of the wind and attempt to isolate its effect on the surface circulation. We therefore shift from HYCOM 1/25° assimilated fields to HYCOM 1/25° free run in 2010. Both HYCOM 1/100° and 1/25° simulations have the same forcing conditions, including the 2010 Navy Operational Global Atmospheric Prediction System winds. The idea is two-fold: first, to allow the submesoscale features resolved by the 1 km resolution model to evolve freely without the disruption of data-assimilation, and second, to isolate the contribution of the surface winds on the continental shelf. Both free runs were initialized in December 2009.

To monitor the circulation on the shelf, a set of 48 particles were launched on the LAF shelf at the beginning of every month from January to September 2010 and advected over 6 months with velocity fields from both HYCOM 1/25° (blue) and HYCOM 1/100° (red) free runs. The link to the animation is: [https://dl.dropboxusercontent.com/u/21001310/animsBOEM/shelfComp.gif](https://dl.dropboxusercontent.com/u/21001310/animsBOEM/shelfComp.gif).

Results show that the models surface velocities in the GoM interior start to diverge within the first two months after the nesting of HYCOM 1/100° at the beginning of the year. However, the circulation on the shelf appears nearly identical, including sudden changes in the direction of the flow and operating in a synoptic pattern characteristic of wind control.
Hovmöller diagrams of the Lagrangian zonal velocities $u_p$ for both HYCOM free runs are plotted in Fig. 2.6 (top left and right panels). Two remarkable features can be observed: the temporal variations of $u_p$ are nearly independent of the particles location on the shelf, and they are very similar in both simulations, in spite of differences in horizontal resolution and model output frequencies. (Model outputs are every 3 hours for 1/100° versus daily for 1/25°). To check whether this trend is realistic, a Hovmöller diagram of the SCULP-II Lagrangian zonal velocities on the shelf was plotted in Fig. 2.6 (lower panel). One can see similar patterns of variation (i.e. weak dependence on the location), which confirms the near-synoptic control of the wind on the direction of the flow.

Figure 2.6: Upper panel: time evolution of the shelf zonal Lagrangian velocities (units in m/s) for HYCOM1/100° (left) and HYCOM1/25° (right) free runs, obtained from a set of 48 particles launched on the LAF shelf every month from January to September 2010, and advected over 6 months. Lower panel: Zonal Lagrangian velocities of the SCULP-II drifter data.
The impact of the higher resolution run on the relative dispersion of the continental shelf in the GoM was examined separately for the LAF shelf starting east of New Orleans, the Louisiana-Texas (LT) shelf west of New Orleans, and the West Florida shelf (WFS), which is located east of the LAF shelf domain. From 5,000 to 10,000 particles were released in those regions. Since the kinetic energy is much lower on the shelf, particle-pairs there take a much longer time to separate than those in the GoM interior. For a limited advection duration (2 to 3 months), this often results in biased-high FSLE estimates at the intermediate and larger scales, obtained from the pairs reaching the GoM interior. The exponential regime at the small scales is already constrained by the positive partition of the Okubo-Weiss parameter. It is therefore preferable to compute \( \lambda(\delta) \) from chance pairs and analyze the impact of the wind on the intermediate scales of 10 to 100 km.

Fig. 2.7 illustrates the behavior of \( \lambda \) for the four launch dates corresponding to each season. The difference between the two free runs at \( \delta < 10 \) km is due primarily to the model resolution constraints: the 4 km simulation cannot resolve features below the 10 km scale while the 1 km does, and the discrepancy is accounted for by the Eulerian strain rate discussed above. Beyond the 10 km scales, we notice that while the Richardson power-law regime is present in all cases, the FSLE curves of both simulations appear to shift to higher or lower values at the same rate in the 20 to100 km scale-range, and that confirms the dominant influence of the wind. The same trend is observed for the LT shelf (Fig. 2.8), with lower values for launch days 91 and 181 (indicating that the particles stayed on the shelves and/or moved inshore), and higher values in the other two launch days 1 and 271. Regarding the WFS (Fig. 2.9), the FSLE-synchronicity of both runs is no longer pronounced, due to the proximity of the Loop Current.

It is unclear what characterizes the relative dispersion at the scales below 10 km, whether small coherent features have time to develop or whether the wind variability and tides generate a more chaotic type of motion. None of the models include the tidal component, for instance. A small-scale relative dispersion experiment on the shelf might help shed some light on this problem.

### 2.3 Submesoscale relative dispersion from GLAD drifter data

#### 2.3.1 GLAD experiment and FSLE estimates

The impact of submesoscale motions on the Lagrangian transport is still being investigated, and recent efforts have been made by CARTHE to analyze submesoscale dispersion measurements from the release of a high number of surface drifters in the northern GoM. The experiment was designed to optimize multi-scale relative dispersion measurements in the vicinity of the Deepwater Horizon (DWH) site where submesoscale features are more likely to emerge from the development of mixed layer instabilities near frontal zones (Özgökmen et al., 2011), such as the edge of the Mississippi River outflow.

The GLAD experiment, which is part of the CARTHE consortium, consisted of more than 300 surface SPOT GPS drifters (cf http://www.findmespot.com released from July 20 to July 31, 2012, at a time of year close enough to the DWH event in 2010. The goal was to quantify the
Figure 2.7: Top panel: the launch domain of the Louisiana-Alabama-Florida (LAF) shelf, superimposed on the 50 m and 100 m isobaths. Middle and lower panels: scale-dependent FSLE ($\lambda(\delta)$) of both free runs from particles launched at 4 different dates on the LAF shelf, and superimposed on the full GoM FSLE. All curves are computed from chance-pairs.
Figure 2.8: Top panel: the launch domain of the Louisiana-Texas (LT) shelf, superimposed on the 50 m and 100 m isobaths. Middle and lower panels: scale-dependent FSLE ($\lambda(\delta)$) of both free runs from particles launched at 4 different dates on the LT shelf, and superimposed on the full GoM FSLE. All curves are computed from chance-pairs.
Figure 2.9: Top panel: the launch domain of the West-Florida shelf (WFS) superimposed on the 50 m and 100 m isobaths. Middle and lower panels: scale-dependent FSLE ($\lambda(\delta)$) of both free runs from particles launched at 4 different dates on the WFS, and superimposed on the full GoM FSLE. All curves are computed from chance-pairs.
scale-dependence of the surface flow from Lagrangian two-point dispersion, and this required high-frequency sampling of position and velocity data in a range of separation scales encompassing the submesoscale and meso-submesoscale boundary (Poje et al., 2013).

The release consisted of several clusters, each launched in a near-simultaneous fractal configuration (cf Fig. 2.10) designed to also maximize the number of available pairs in the submesoscale range (100 m to 10 km) and to capture all details of transport, thanks to a high GPS accuracy and sampling frequency (4 to 5 meter std error and 5-minute coordinate transmission). The S1 and S2 clusters were composed of 10 nodes in an S-configuration (Fig. 2.10, left panel). Each node was composed of 3 triplets 500 m apart, themselves composed of 3 smaller triplets with minimum distances of 100 m (Fig. 2.10, right panel), amounting to a total of 90 drifters per cluster. S1 was released at the DWH site on July 22, while S2 was released above the De Soto canyon straddling a front on July 26. The T1, L1, and L2 clusters were also deployed in the region in a fractal configuration of 27 and 63 clusters, with minimum relative distances of 200 m between the smallest triplets (Fig. 2.11 upper panel).

As illustrated in Fig. 2.11, the S1 and S2 clusters resulted in radically different patterns of dispersion, with S2 spreading along what appears to be an outflowing branch, and S1 evolving slowly along the shelf break with very little dispersion. However, all clusters yield nearly identical scale-dependent FSLEs for the first 3-week trajectories (Fig. 2.12 top panel).

The curves appear to be generic with a local regime that follows a power-law of \(\delta^{-0.5} - \delta^{-0.6}\). It is close to the \(\delta^{-2/3}\) Richardson regime, and implies that the dispersion at the scales \(\delta \leq 10\) km is controlled by SMS features of the same scales. \(\lambda(\delta)\) of synthetic trajectories from both HYCOM
Figure 2.11: 4-day trajectories of the different drifter groups released by the GLAD cruise including SPOT, flat surface drifters, NOAA, meteorological (MET) drifters and drifters released by the US Coast Guard. The S1, S2, T1, L1 and L2 clusters were released in a fractal configuration.

1/100° free run for the full GoM, and from the Navy Coastal Ocean Model (NCOM) assimilation run are also plotted to highlight the generic differences between models and observations. NCOM was also run at NRL with a 3-km resolution and particles were launched with the GLAD’s initial conditions for each cluster.
Figure 2.12: Top panel: $\lambda(\delta)$ of each GLAD drifter cluster for the first 3 weeks after their release. Superimposed are the FSLE of the NCOM (3km resolution) trajectories with same initial conditions, and the SPOT GPS noise-contribution to $\lambda(\delta)$ for a 5-min sampling interval. Bottom panel: $\lambda(\delta)$ of the S1 and S2 GLAD 1-week trajectories computed from chance pairs, and their 1-hour filtered counterparts.
A comparison of model and GLAD FSLEs displays high discrepancies in the rates of relative dispersion up to about 10 km: \( \lambda(\delta) \) reaches values of 10 day\(^{-1}\) at 100 m scales, down to 2 day\(^{-1}\) at 1 km and around 0.8 day\(^{-1}\) at 10 km. (A more in-depth analysis is in Section 2.3.2). As expected, both models impose an exponential regime with \( \lambda_{\text{max}} \) values set by the average stretching rate of the simulated flow field. Note that with the exception of the L1,2 cluster, the NCOM Lyapunov exponent ranges below the HYCOM 1/100\(^0\)\( \lambda_{\text{max}} \), which is consistent with the coarser grid-resolution. The only instance where a change in the GLAD FSLE occurs is during the first week with the S2 cluster (bottom panel of Fig. 2.12). \( \lambda(\delta) \) of S1 and S2 is computed from chance pairs to produce statistically reliable estimates at both small and large scales, and displays lower values for S2 at scales below 3 km and a flatter slope, that is mostly likely caused by the exponential rate of separation along the outflowing branch mentioned above. Beyond the 10 km scales, the S2 pairs separate faster than S1 and \( \lambda(S2) \) becomes higher than \( \lambda(S1) \).

### 2.3.2 Impact of subsampling and measurement noise on the scale-dependent FSLE

The impact of the SPOT GPS position error on the FSLE, which is more likely to be biased at the small-scales, was also investigated. Recent work has been done on the impact of noise and subsampling on relative dispersion measurements (Haza et al., 2013), which has led to the investigation of time-cumulative dispersion, and how it can be inferred from simple filtering and subsampling of the drifter trajectories.

The case of zero-cumulative relative dispersion is a noise in the GPS position error with a std of \( L_K \) and a sampling time of \( Dt \). If the noise dominates the relative dispersion at a certain range of scales, then the FSLE will follow a spurious \( \delta^{-1} \) regime and can be approximated by the expression:

\[
\lambda(\delta) \sim \frac{\log(\alpha)}{(\alpha - 1) Dt} \frac{L_K}{\delta}.
\]

Thus, subsampling the trajectories is equivalent to increasing \( Dt \), and \( \lambda(\delta) \) is reduced at each scale according to the expression above. The same trend is observed by low-pass filtering the trajectories from a moving average with a window \( T \equiv Dt \). As we keep increasing \( DT \), the noise signal lowers to the FSLE level of the real signal, which always has a positive cumulative dispersion. This simple process allows for a more reasonable estimate of the real FSLE (albeit from the subsampled/filtered trajectories).

How much the FSLE of the real signal is reduced is found to depend in part on the monotonic increase of the averaged relative distance of the drifter-pairs (or time-cumulative dispersion, cf Fig. 2.13, bottom panel), and in part on the time-scale of the flow features controlling the FSLE at a given scale. The FSLE of the noise meanwhile provides a reference for the absence of cumulative dispersion.

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Figure 2.13: Upper panel: $\lambda(\delta)$ of the 48-hour low-passed (sub-sampled) trajectories for the GLAD data and numerical models. Lower panels: Relative distance $D(t)$ of two drifter-pairs with equal raw $\tau$ estimate but differing time-cumulative dispersion.

This new approach has led to a more in-depth analysis of the GLAD relative dispersion: although the four clusters of GLAD (S1, S2, T1, L1,2) have different types of time-dependent relative dispersion, the scale-dependent FSLE is pretty much the same, implying that a common factor is
causing the generic local dispersive regime at the submesoscales. The GPS uncertainty was investigated by applying filters of different strengths to the trajectories. A 1 hour low-pass filter is enough to show the robustness of $\lambda(\delta)$ and confirms that the high values and non-local $\lambda \sim \delta^{-0.5}$ regime are indeed unrelated to position errors (Fig. 2.12 bottom panel).

Another common factor to all clusters is the presence of diurnal oscillations (inertial and possibly other wind and tidal components). The trajectories were then filtered with a 48h low-pass window to remove the oscillations, or subsampled with a $Dt=48$ hrs, and the results are displayed in the top panel of Fig. 2.13 for the S1-cluster. Unlike the HYCOM or NCOM trajectories with same initial conditions, $\lambda(\delta)$ is reduced dramatically in the absence of diurnal oscillations. Note however that the curve of the filtered GLAD trajectories is still substantially higher than the model’s FSLE of their filtered counterpart up to $\delta=10$ km, indicating that cumulative dispersion is generated by features at those scales that the models cannot resolve.

Those findings are significant for Lagrangian parametrization in that LSMs add some form of diffusivity, which tends to increase monotonically the relative dispersion as a function of time by steadily injecting diffusion. If one attempts to match the scale-dependent FSLE of in-situ trajectories by addition of an LSM to daily outputs of a numerical model for example, then it is better to consider the FSLE of the 24 hr filtered trajectories for reference, since the values are closer to the time-cumulative dispersion. It is still unclear what impacts the 10 to100 m SMS features have on the dispersion.
3 LSM-3 for the GoM interior

The Lagrangian parameterization for the GoM interior seems straightforward, since the change in the scale-dependent FSLE is very similar to what we found for the Gulf Stream’s relative dispersion: λ differs from the smallest scales to the kink (the change in slope shown in Fig. 1.1) at twice the Rd marking the regime transition from non-local to local mesoscale dispersion. The third Lagrangian stochastic model (LSM-3) appears then to be the right parameterization to implement from a 4 km (or coarser, yet eddy permitting) resolution OGCM.

3.1 LSM-3 free HYCOM runs

3.1.1 Flow decomposition

Both the temporal moving average (L) and spatial Gaussian weighted average (G) are considered as low-pass filters on the HYCOM 1/25° velocity fields. Fig. 3.1 (upper panel) shows the impact of the different filters for three different strengths on the scale-dependent finite scale Lyapunov exponent (FSLE) λ(δ). The filter strengths were selected to satisfy the flow decomposition requirements in the context of scale-dependent relative dispersion. The model exhibits an exponential plateau around 0.25-0.3 day\(^{-1}\) and a transition to the local regime around δ=50 km. Each filter lowers the average hyperbolicity from 0.2 day\(^{-1}\) for filters G=1.5 grid-size and L=5 days, down to 0.15 day\(^{-1}\) for G=5 grid-size and L=21 days. All filtered velocities have the same relative dispersion at the scales δ >2Rd as the model, a requirement for the LSM-3 implementation.

The corresponding statistical parameters are obtained from computing the autocovariance functions at different launching times, and displayed in Fig. 3.1, lower panel. In both zonal and meridional directions, the velocity residuals of the Gaussian filter test beds tend to have smaller velocity fluctuations and longer decorrelation time scales than their temporal average counterpart for similar FSLEs of the low-passed velocities. A summary of all optimal parameters (including the continental shelf) is given in Table 3.1.

3.1.2 LSM-3 results for scale-dependent FSLE λ(δ)

The LSM-3 was first implemented with HYCOM 1/25° by setting the “real”/”target” parameters to twice the model parameters, i.e.: (σ, τ) = (2σ, 2τ). Results from the parameterized trajectories reveal an enhancement of the FSLE plateau for all cases, with λ\(_{max}\) between 0.3 and 0.4 day\(^{-1}\), and a smoother transition to the large-scale local regime, the latter being relatively unchanged (cf Fig. 3.2). Note that the experiments with strong low-pass filters, such as G5 and L21 (in red) result in higher FSLE values than those with weaker filters, such as G1.5 and L5 (in blue). Indeed, strong filters leave more kinetic velocity residuals, which are the flow components modified by the LSM-3 formulation.
Figure 3.1: Upper panel: $\lambda(\delta)$ of GoM trajectories from particle ensembles advected by low-passed velocity fields satisfying the flow-decomposition requirements for the LSM-3. Blue, green and red curves are for weak, medium and strong filters, respectively. The model curve is in black. G and L (or LPt) decompositions correspond to Gaussian and local temporal filters, respectively. Lower panel: corresponding model Markov-1 parameters $\sigma_m$ versus $\tau_m'$ of the meridional component.
Table 3.1: Optimal model \((\sigma^{u}_m, \tau^{u}_m, \sigma^{v}_m, \tau^{v}_m)\), target/real \((\sigma_r, \tau_r)\), and LSM-3 corrected \((\sigma^{u}_c, \tau^{u}_c, \sigma^{v}_c, \tau^{v}_c)\) Markov-1 parameters applied to HYCOM 1/25° free run for different filters. The numbers in parenthesis are the corrected-model parameter ratios (to be compared with the real-model ratios).

<table>
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<th>(\tau^{u}_m) (day)</th>
<th>(\sigma^{v}_m) (cm/s)</th>
<th>(\tau^{v}_m) (day)</th>
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<td></td>
<td></td>
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<td>(1.5, 1.5)</td>
<td>15.5</td>
<td>(1.6, 1.2)</td>
</tr>
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<td>1.33</td>
<td>8.7</td>
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<td>12.3</td>
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<td>(1.25, 1.65)</td>
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<td>L11</td>
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<td>1.83</td>
<td>4.3</td>
<td>1.5</td>
<td>6.9</td>
<td>(2, 2)</td>
<td>8.0</td>
<td>(1.8, 1.67)</td>
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<td>2.5</td>
<td>1.17</td>
<td>(2, 2)</td>
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<td>(2.0, 1.8)</td>
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<td>(1.5, 1.5)</td>
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\(\sigma^{u}_m\) - model zonal velocity fluctuation std  
\(\sigma^{v}_m\) - model meridional velocity fluctuation std  
\(\tau^{u}_m\) - model zonal decorrelation time-scale  
\(\tau^{v}_m\) - model meridional decorrelation time-scale  
\(\sigma^{u}_c\) - corrected zonal velocity fluctuation std  
\(\sigma^{v}_c\) - corrected meridional velocity fluctuation std  
\(\tau^{u}_c\) - corrected zonal decorrelation time-scale  
\(\tau^{v}_c\) - corrected meridional decorrelation time-scale  
\(\sigma_r, \tau_r\) - real/target velocity fluctuation std and decorrelation time-scale for both directions

Direct comparison with the target FSLE (HYCOM 1/100° free run) indicates that doubling \((\sigma_m, \tau_m)\) underestimates the FSLE levels of HYCOM 1/100° with \(\lambda_{max} \sim 0.5 \text{ day}^{-1}\) regardless of which filter we choose, although the strongest filters (G5 and L21) can reproduce the target FSLE down to 20 km. However, increasing \(\sigma_m\) by a factor of 3 appears to further increase \(\lambda(\delta)\) and reaches the FSLE level of the target run at least in the intermediate scales. In that instance, only the relatively small filter strengths of L5, G1.5 and G3 are considered (Fig. 3.3, lower panel); the std of velocity fluctuations must be small enough (~4 cm.s\(^{-1}\)) to be tripled while remaining below the order of magnitude of the typical GoM velocity scales, to preserve the Lagrangian mesoscale pathways.
3.1.3 LSM-3 results for the relative dispersion $D^2(t)$

The time-evolution of the averaged relative dispersion $D^2(t)$ is plotted in Fig. 3.4. The LSM-3 experiments where the model parameters are doubled (top panel) are superimposed on the model dispersion curve (in black). The HYCOM1/100° curve is also added (brown curve), but only as reference for what values to expect. When computed from thousands of particle pairs released over the entire domain, the relative dispersion generally increases with the model resolution, as is the case here.

The LSM-3 is seen to enhance the relative dispersion at all times, including the model’s exponential regime in the first few days, and followed by a time power-law of $t^3$, characteristic of mesoscale control. The exponential regime of the parameterized trajectories (Fig. 3.4, top panel) is still below the level of HYCOM 1/100°, as is expected from their Lyapunov Exponents. The LSM-3 curves then reach the level of the HYCOM 1/100° run after about 10 days; they even overshoot it in the strong filter cases of G5 and L21 in the second month (Fig 3.4). This result is not surprising, considering that the Markov-1 parameters are known to increase the absolute dispersion by increasing $\sigma$ and/or $\tau$ (Haza et al., 2007).
Figure 3.3: $\lambda(\delta)$ of LSM-3 trajectories for the weak filters L05 (upper left), G1.5 (upper right) and moderate filter G03 (lower panel), including the expected $(\sigma_r, \tau_r) = (3\sigma_w, 2\tau_w)$ parameter-sets. HYCOM 1/25° and 1/100° curves are also included.

$D^2(t)$ is also computed for the $(3\sigma_w, 2\tau_w)$ experiments and plotted in the lower panel of Fig. 3.4. Note that we observe an even better fit to the reference run, although the exponential regime is still underestimated. In this case, only the G3-set is seen to overshoot the HYCOM 1/100° run after 10 days, which was suggested by the overshooting kink in its FSLE counterpart.

3.1.4 Impact of LSM-3 on Lagrangian accelerations and frequencies

The impact of LSM-3 on the Lagrangian frequencies and accelerations were examined since both are sensitive to Lagrangian stochastic models as well as SMS motions. The trajectory ensembles were obtained from about 7,000 particles launched at day 91, 2010 in a 1 km-triplet configuration and advected over 3 months. This period of the model free run was fairly representative of the rest of the year.
Figure 3.4: Upper panel: $D^2(t)$ of the LSM-3 trajectories corresponding to the filters displayed in Fig. 3.1 after doubling the model parameters, superimposed on the model and reference HYCOM1/100° curves. Lower panel: $D^2(t)$ of the LSM-trajectories with expected $(\sigma_r, \tau_r) = (3\sigma_m, 2\tau_m)$ for the filters L05, G15 and G03.

The frequency spectra of the model and target run had significant differences (Fig. 3.5, upper panel). The power spectrum density (psd) of the target run was higher than the model at $\nu > 0.1$ day$^{-1}$, which originated from two factors: the ability of HYCOM 1/100° to resolve submesoscale motions, and a higher model output frequency of 3 hours (versus daily in our case for HYCOM 1/25°). Here again, the LSM-3 of all parameters tested with $(\sigma_r, \tau_r) = (2\sigma_m, 2\tau_m)$ were shown to enhance the frequency densities of the model at the range 0.05-2 day$^{-1}$, yet not to the level of the target run.
Figure 3.5: Single particle statistics of the LSM-3 cases. Upper panel: Lagrangian frequency spectrum of the LSM-3 trajectories superimposed to both HYCOM cases. Lower panel: Lagrangian accelerations of different parametrized trajectories, including targeted $(2\sigma_m, 2\tau_m)$ and $(3\sigma_m, 2\tau_m)$. The enhancement in the higher frequencies and Lagrangian accelerations is still below the HYCOM1/100° levels.
Even more so, the Lagrangian acceleration psd showed again dramatic differences between both HYCOM free runs, with much higher particle accelerations for the 1 km resolution run, and the inability of the LSM-3 to match them (Fig. 3.5, lower panel). Nonetheless, the LSM-3 modified the model’s psd. The more energetic \((3\sigma_m, 2\tau_m)\) sets showed a significant improvement in the particle accelerations, yet still far from the target run. Note that additional tests (not shown) on the HYCOM 1/100° output frequency have confirmed that increasing the time-interval between outputs from 3 hrs to 24 hrs still yields substantial differences in the Lagrangian acceleration psd of the HYCOM runs. It confirmed that other factors besides the higher output frequency of the reference run account for the psd discrepancy.

Unlike previous tests on the Gulf Stream parameterization where some parameter sets were overestimating the Lagrangian accelerations and frequencies, these results indicated that no additional filtering on the model velocities was needed in this setting.

### 3.1.5 Qualitative dispersion comparisons

Since the circulation in the Gulf of Mexico is strongly constrained by the surrounding coast and the Loop current system, evaluating the performance of a Lagrangian stochastic model with statistical relative dispersion or cloud dispersion is not always straightforward. A complementary approach is to do a qualitative analysis of the ensemble particle dispersion, similarly to tracer dispersion. We selected a region that is part of the GoM interior, but still close enough to the continental shelf and Mississippi River outflow to capture different flow dynamics. We launched 2,601 particles 4 km apart on a regular array. The particles were advected for two months in HYCOM 1/25°, starting at day 20.

Lagrangian parameterization was then included and the evolution of particles for different cases as well as the model was plotted after 30 and 50 days of the launch date in Figs. 3.6 and 3.7, respectively. The particle patch evolution from the model (upper left panel) illustrates the existence of an anticyclonic eddy south of New Orleans and filamentation due to horizontal stretching as particles disperse in the GoM interior. Four LSM-3 sets are presented with doubling of the model parameters: two weak flow-decompositions with G1.5 and L5 filters (middle and lower left panels), and two strong ones with G5 and L21 filters (middle and lower right panels). An LSM-1 (uncorrelated with velocity fluctuation stds of 500 m/2hr) was also implemented as reference to the more classic stochastic model (upper right panel).

The LSM-1 and LSM-3 operate differently on the dispersion, in that the former tend to diffuse the edges and filaments of the mesoscale field, even though the random kicks are mild enough to preserve the transport barriers. This translates into thicker and grainier filaments where the particles concentrate. On the other hand, the LSM-3 tends to accelerate the filamentation and/or stretching. This tendency is more pronounced with stronger low-pass filters for the flow decomposition (i.e., G5 and L21). Fifty days after launch, the LSM-3 clusters of the strong filter cases reach dispersion levels that are significantly higher than those obtained by advection from the model alone and by the LSM-1 experiment. The sharpness of the model transport barriers is maintained, and the particles manage to cover a higher number of their pathways. On the other hand, the LSM-1 tends to cover the same mesoscale pathways as the model. This limitation has
been consistently observed in other configurations and appears to be related to this type of random-walk implemented with a random kick of about 10% mesoscale displacement per 1 to 2 hr time step (i.e., corresponding to a velocity fluctuation std of 10% the mesoscale typical velocity scale).

Figure 3.6: Particle positions 30 days after launch for the model (upper left), the LSM-1 or random walk (upper right), and the LSM-3 cases with the targeted \((2\sigma_m, 2\tau_m)\)-cases for low-pass filters G1.5 (middle left), G5 (middle right), L5 (lower left) and L21 (lower right). NB: the notation “2s,2t” in the figure corresponds to \((2\sigma_m, 2\tau_m)\).

Note that the coverage of particles from the LSM-3 experiments reached the reference level, in spite of the fact that the modified Lagrangian accelerations by the term \(\partial\eta/\partial t\) in Eq. 14 was still underestimated by the stochastic model. However, the missing component \(\eta\) acts selectively on the regions of highest shear and strain. These include the hyperbolic regions which increase the relative dispersion dramatically.
3.1.6 Impact on tracer dispersion

To verify how a tracer would behave with the LSM-3, one can release a patch of particles at a very high density and monitor its evolution for a given duration. How the patch evolves is also sensitive to the grid-resolution. We launched more than 100,000 particles on a small square of 40 by 40 km centered on the DWH site to verify how the LSM-3 would behave and we advected them for 1 month in both HYCOM models. The launch was repeated monthly to get a statistically relevant estimate of the spatial coverage, as well as the distortion pattern of the tracer. It is possible in this context to compare the performance of the two free runs, if one considers all the launches combined into one statistical ensemble. The results are plotted in Fig. 3.8.
Figure 3.8: Patch releases on the DWH site, composed of 103,041 particles (top-left). Patch position after 1 month for the reference run (top-right), the model (middle-left), and for the LSM-3 cases L11-(2σ, 2τ), G03-(2σ, 2τ), and L05-(3σ, 2τ). Each color corresponds to a different launch/realization of the statistical ensemble in 2010: days 61 (black), 91 (green), 121 (red), 151 (blue), 181 (magenta), 211 (cyan), 241 (yellow), and 271 (brown). Note: the dates are irrelevant, since the tracer distributions depend on the mesoscale field of a free run.

The patch is stretched into filaments and/or describes loops if it is caught in an eddy, as predicted by dynamical system theory. There is nevertheless a difference between the two models, in that the patch advected by HYCOM1/100º displays more convoluted patterns and covers a wider portion of the GoM. The experiments are then repeated with 3 other LSM-3 parameterized sets of L11-(2σ, 2τ), G03-(2σ, 2τ), and L05-(3σ, 2τ). The LSM-3 was also applied to the shelf with a weak filter (G15). The results in Fig. 3.8 show a net improvement of the model patterns and coverage, which can be quite similar to the high-resolution run. Although there might be some indication of overshooting with the L11 filter, the LSM-3 patches become more convoluted.
while still stretching. This effect is even more pronounced with the third experiment, since the amplitudes of the model velocity fluctuations are tripled. Note also that the natural transport barrier separating the West Florida shelf (forbidden zone) from the GoM interior is generally preserved for all the LSM-3 experiments, while the patch tends to intrude on the northern portion of the WFS in a way that is closer to the HYCOM1/100° reference run than the HYCOM1/25°.

The same experiment is repeated on the Western side of the GoM, and illustrated in Fig. 3.9. Here again, we observe a wider coverage of the area by the reference run HYCOM1/100° and more convoluted patterns marking the presence of small-scale features. The LSM-3 parameterization is seen to reduce the discrepancy of the pattern signatures and slightly increase the coverage, which remains mostly on the western side of the basin.

Below are links to a list of animations from the experiments described in this section. These animations illustrate the impact of the LSM-3 on the evolution of tracers, and how they compare to the model itself. Particle-based tracers are launched at the DWH or Western GoM (WES) sites (see Figs 3.8 and 3.9), and are advected by either HYCOM1/25° (“model”), HYCOM1/100°, or HYCOM1/25° combined with the LSM-3. Launch numbers correspond to different dates in 2010. The LSM-3 is characterized by the low-pass filter and statistical parameters of the GoM interior, followed by those of the shelf. These animations are also included in the DVD attached to this report under the directory `animBOEM`.

DWH HYCOM1/25° patch launches 2 through 5:
https://dl.dropboxusercontent.com/u/21001310/animBOEM/mod-02.gif
https://dl.dropboxusercontent.com/u/21001310/animBOEM/mod-03.gif
https://dl.dropboxusercontent.com/u/21001310/animBOEM/mod-04.gif
https://dl.dropboxusercontent.com/u/21001310/animBOEM/mod-05.gif

DWH HYCOM1/100° patch launches 2 through 5:
https://dl.dropboxusercontent.com/u/21001310/animBOEM/1km-02.gif
https://dl.dropboxusercontent.com/u/21001310/animBOEM/1km-03.gif
https://dl.dropboxusercontent.com/u/21001310/animBOEM/1km-04.gif
https://dl.dropboxusercontent.com/u/21001310/animBOEM/1km-05.gif

DWH comparison model (blue) / LSM-3 (red) [G03-(2σ, 2τ)-G15-(1.5σ, 1.5τ)] patch launches 2 through 4:
https://dl.dropboxusercontent.com/u/21001310/animBOEM/DWHcomp-02.gif
https://dl.dropboxusercontent.com/u/21001310/animBOEM/DWHcomp-03.gif
https://dl.dropboxusercontent.com/u/21001310/animBOEM/DWHcomp-04.gif

WES comparison model (blue) / LSM-3 (red) [G03-(2σ, 2τ)-G15-(1.5σ, 1.5τ)] patch launches 4 and 8:
https://dl.dropboxusercontent.com/u/21001310/animBOEM/WEScomp-04.gif
https://dl.dropboxusercontent.com/u/21001310/animBOEM/WEScomp-08.gif
Figure 3.9: Patch releases in the WES site, composed of 103,041 particles (top-left). Patch position after 30 days for the reference run (top-right), the model (middle-left), and for the LSM-3 cases L11-(2σ, 2τ), G03-(2σ, 2τ), and L05-(3σ, 2τ). Each color corresponds to a different launch/realization of the statistical ensemble.

3.1.7 LSM-3 over extended periods of time

The Patch experiments were extended to 6 months in an attempt to find out whether particles tend to cross from the east side to the west side of the GoM, and vice-versa. Some interesting differences were observed between the two models: particles in HYCOM 1/100° more easily travel to the other side of the Gulf. This is much less the case with HYCOM 1/25°, where particles have a preference for the east side of the GoM, no matter where they are launched. The LSM-3 did not succeed in changing that tendency, although it managed to enhance the dispersion.
of the particles crossing the basin. Since the tests were conducted for less than a year, it is unclear how representative those trends are.

### 3.1.8 SMS seasonality

A short note on the Lagrangian parameterization in the context of SMS seasonality: since the SMS are less active in spring and summer, it is necessary that the statistical parameters be chosen such that they do not exceed the FSLE level computed in the HYCOM1/100° free run during those times. For instance, the $\lambda_{\text{max}}$ levels in winter in Fig. 2.3 are seen to be around 0.6 day$^{-1}$ and down in summer by as low as 0.35 day$^{-1}$. The Lyapunov exponent in HYCOM1/25° is $\sim0.25$ day$^{-1}$. The LSM-3 with target-parameters being twice the model parameters reach levels of 0.3-0.4 day$^{-1}$, and are therefore within the threshold of the high resolution run in spring/summer. However, LSM-3 sets where the velocity fluctuation amplitudes are tripled can exceed the summer values, though not by much.

### 3.1.9 Sensitivity to the initial value of the missing component $\eta_0$

With regard to the sensitivity of the initial choice of $\eta_0$ (i.e., the value of $\eta$ at the time of launch), repeated experiments were conducted where $\eta_0$ is assigned a different value for each launch of the particle ensemble. The values of $\eta_0$ are selected arbitrarily, yet their range is within the variance of the missing component (as described in the Eq. 16). Fig. 3.10 illustrates the slight variations in the tracer distribution after 1 month advection from the DWH site with the G03-$\left(2\sigma, 2\tau\right)$ parameter set. These results highlight the weak sensitivity of the LSM-3 to the initial value of $\eta(t)$, which is also the only instance when a random or arbitrary value (within the range defined by the model and target parameters) is assigned. The main impact of the LSM-3 on tracer distributions appears to be mostly determined by the flow-decomposition and rates of change of the Markov-1 parameters. Nonetheless, increasing an ensemble of parameterized trajectories by repeated launches with several $\eta_0$s improves the statistical relevance of a Lagrangian prediction.

### 3.2 LSM-3 with altimetry and drifter data

The goal here was to test a Lagrangian parametrization in a real time setting by using a velocity field as close as possible to the in-situ surface velocities during the GLAD experiment. Tests done by collaborators from CARTHE indicated some issues with the assimilation of altimetry, leading to the conclusion that it was preferable at the time to use the geostrophic velocities from the satellite-derived sea surface heights. Here, we used the geostrophic velocity field generated on a 9 to 10 km regular grid from the AVISO altimetry product, that includes a non-divergent flow near the coastline (work done by M. Iskandarani and J. Mensa, oral communication, 2013, unpublished data).
Figure 3.10: Patch positions 1 month after their releases from the DWH initial conditions. The different colors correspond to a different $\eta_0$ for each launch.

Qualitative comparison of GLAD and AVISO trajectories indicate that the drifters tend to follow the mesoscale pathways of the geostrophic velocities, as can be seen from animation (1) at the end of this section. The main discrepancies occur on or near the shelf. In general, we can assume that the mesoscale field of the GoM interior is well approximated by the AVISO geostrophic velocities, making it an ideal test-bed for SMS Lagrangian parameterization. Because of the irregular satellite tracks, interpolations in both space and time are required to produce sea-surface height values on a regular array. This leads to very smooth geostrophic velocities on a coarse resolution, and thus low stretching rates of dispersion. It is confirmed by the scale-dependent FSLE of AVISO synthetic trajectories plotted in Fig. 3.11 (upper panel), which shows a weak exponential regime at 0.2 day$^{-1}$, extending up to 100 km (lower than HYCOM 1/25°’s $\lambda_{\text{max}}$). The figure also displays a big discrepancy with the FSLE of HYCOM1/100°.

The LSM-3 is implemented on the geostrophic velocities by using strong filters, such as the temporal moving averages of L21 and L11, since applying a low-pass filter on a smooth field leaves a very limited turbulent component. Indeed, doubling the AVISO parameters (cf Table 3.2) results in only marginal improvement of the Lyapunov exponent around 2.5 day$^{-1}$. However, the enhancement of the curve at the intermediate scales of 50 to 200 km is significant enough to increase the relative dispersion. Following the selection of optimal parameters, a patch of 7,103 particles was launched on July 28, 2012 (right after the S2 launch) in a rectangular domain encompassing the location of the GLAD S1 and S2 clusters (Fig. 3.11 lower panel).

The evolution of the patch is displayed in Figs. 3.12 to 3.14, where it is advected either by the geostrophic velocities alone (upper left panel), or combined with a strong random walk of $L_k=1$ km/2 hours (upper right panel), or with the LSM-3 sets of L11d-(2$\sigma$,2$\tau$) and L21d-(2$\sigma$,2$\tau$) (lower panels).
Figure 3.11: Top panel: $\lambda(\delta)$ of the AVISO trajectories for the whole GoM (blue curve), and of two LSM-3 sets of parameterized trajectories. Superimposed is the FSLE of the reference run (dashed line). Bottom panel: Release of a rectangle patch of 7,108 particles on July 28, 2012 to be advected by the AVISO velocities. Red dots are the GLAD drifter positions after being low-passed filtered with a 48 hr moving average.

Table 3.2: Markov-1 optimal zonal and meridional statistical parameters of the AVISO geostrophic velocity field in the GoM interior for the corresponding filters L11 and L21.

<table>
<thead>
<tr>
<th>Filter</th>
<th>$\sigma_u^A$ (cm/s)</th>
<th>$\tau_u^A$ (days)</th>
<th>$\sigma_v^A$ (cm/s)</th>
<th>$\tau_v^A$ (days)</th>
</tr>
</thead>
<tbody>
<tr>
<td>L21</td>
<td>4.0</td>
<td>1.42</td>
<td>4.8</td>
<td>1.42</td>
</tr>
<tr>
<td>L11</td>
<td>2.4</td>
<td>1.2</td>
<td>3.1</td>
<td>1.1</td>
</tr>
</tbody>
</table>

1 $\sigma_u^A$ zonal velocity fluctuation std of AVISO
2 $\tau_u^A$ zonal decorrelation time-scale of AVISO
3 $\sigma_v^A$ meridional velocity fluctuation std of AVISO
4 $\tau_v^A$ meridional decorrelation time-scale of AVISO
Note that for all experiments, the dark blue patch is stretched along the “tiger-tail” south of De Soto Canyon in August 7 (Fig. 3.12), and a significant portion describes the cyclonic circulation in August 27 (Fig. 3.13). Yet the particles in the LSM-3 experiments have dispersed much more than the geostrophic velocities and the LSM-1 by borrowing the neighboring pathways of the AVISO velocity field. This is confirmed by many GLAD drifters moving along the stretching lines of the LSM-3 clustered particles (cf animations below). Note that in September 17 (Fig. 3.14), most of the GLAD drifters are surrounded by particles of the LSM-3 experiments, while both the AVISO and LSM-1 particles have not covered the area occupied by the drifters.

This experiment demonstrates the ability of the LSM-3 to substantially improve tracer prediction problems, even with an approximate flow field, provided that the mesoscale features are represented with enough accuracy.

Below are the links to animations of section 3.2 experiments. The FSLE-map in animation (1) displays the attracting LCS, and their role as transport barriers is evidenced by the AVISO 2-day tails. The superimposed GLAD 48 hr low-passed tails illustrate the areas where AVISO and the in-situ velocities match, versus where they differ. The specifics of the animations (2) through (4) are described in Figure captions 25 through 28, and show how the LSMs affect the evolution and coverage of the AVISO tracer. These animations are included in the DVD attached to this report under animsBOEM.

(1) AVISO backward FSLE maps (ridges in gray scales) and AVISO 2 days tails (in black), with GLAD 48h low-passed 2-day trajectories (in red):
https://dl.dropboxusercontent.com/u/21001310/animsBOEM/avisoFs1bGlad.gif

(2) GLAD and AVISO patch:
https://dl.dropboxusercontent.com/u/21001310/animsBOEM/box-Glad.gif

(3) GLAD (in red) drifter low-passed positions and AVISO+LSM-1 (in blue) patch:
https://dl.dropboxusercontent.com/u/21001310/animsBOEM/box-RW0Glad.gif

(4) GLAD (in red) drifter low-passed positions and AVISO+LSM-3 (L11, in blue) in blue:
https://dl.dropboxusercontent.com/u/21001310/animsBOEM/box-2s2tLP11dGlad.gif

(5) GLAD (in red) drifter low-passed positions and AVISO+LSM-3 (L21, in blue):
https://dl.dropboxusercontent.com/u/21001310/animsBOEM/box-2s2tLP21dGlad.gif
**Figure 3.12:** Evolution of the patch (in blue) advected by the AVISO geostrophic velocities (upper left panel) and combined with three LSM cases in August 7, 2012. Upper right panel: a strong LSM-1 random walk ($L_K=1$ km), lower left: LSM-3 of (LP11d-2σ, 2τ) and lower right panel: a stronger LSM-3 set of (LP21d-2σ, 2τ). The GLAD 48h filtered drifter data is plotted in red.
Figure 3.13: Evolution of the patch (in blue) advected by the AVISO geostrophic velocities (upper left panel) and combined with three LSM cases in August 27, 2012. Upper right panel: a strong LSM-1 random walk ($L_x=1$ km), lower left: LSM-3 of (LP11d-2σ, 2τ) and lower right panel: a stronger LSM-3 set of (LP21d-2σ, 2τ). The GLAD 48h filtered drifter data is plotted in red.
Figure 3.14: Evolution of the patch (in blue) advected by the AVISO geostrophic velocities (upper left panel) and combined with three LSM cases in September 17, 2012. Upper right panel: a strong LSM-1 random walk ($L_{x}=1$ km), lower left: LSM-3 of (LP11d-2σ, 2τ) and lower right panel: a stronger LSM-3 set of (LP21d-2σ, 2τ). The GLAD 48h filtered drifter data is plotted in red.
4 LSM for the continental shelf

4.1 LSM results for the relative dispersion metrics, $\lambda(\delta)$ and $D^2(t)$

A set of about 2,500 particles was launched in a triplet configuration 1 km apart over the GoM’s LT and LAF shelves to compute statistical metrics of relative dispersion from two-month trajectories. The launches are repeated twice for days 1, 90 and 181, and the results are displayed in Fig. 4.1, after removing any portion of trajectory over depths exceeding 100 m to minimize the impact of the GoM interior. As expected, the resulting scale-dependent FSLE (computed this time from original pairs) on the shelf is substantially lower than the full GoM FSLE (upper left panel), since the kinetic energy is weaker on the shelf. For HYCOM 1/25°, $\lambda_{max}$ ranges between 0.1 and 0.2 day$^{-1}$ and for HYCOM 1/100°, $\lambda_{max}$ is between 0.2 and 0.6 day$^{-1}$, with the highest values approaching the mesoscale level for each simulation. Additionally, the lack of a kink at 100 km (as shown in Figure 1.1) is an indication that the mesoscales of the GoM interior do not dominate the relative dispersion on the shelf. The time-dependent relative dispersion curves for launch day 1 (upper right panel of Fig. 4.1) show that, after two months, particles initially 1 km apart reached distances of about 10 km only, which is at least an order of magnitude lower than typical estimates for the GoM interior. The higher-resolution run yields slightly higher dispersion estimates than HYCOM 1/25°, yet within the same range of 10 to 50 km. Deviations from these trends are observed in the other launches (lower panels), where the exponential regime and/or maximum Lyapunov exponent reach the mesoscale level, indicating the presence of mesoscale features near the shelf.

In summary, the circulation on the shelf is not mesoscale controlled, and the wind forcing appears to dominate the characteristics of the flow at the surface. Both weak kinetic energy and wind-induced polarized tendency of the surface circulation lead to weak relative dispersion estimates independently of the horizontal resolution of the models.

Since the relative dispersion of both HYCOM 1/25° and HYCOM 1/100° is small, including their differences, the classic random walk and random flight models (LSM-1 and 2) are more likely to perform on the shelf than in the GoM interior. The LSM-1 and LSM-2 are known to generate a highly local regime in the SMS, evidenced by the $\lambda(\delta)$ curves describing $\delta^{-1}$ - $\delta^{-0.7}$ power-laws (Figs. 4.2 and 4.3, upper panels), which are a departure from the non-local regime of the numerical models.
This leads to an overestimation of $\lambda$ at the scales below 10 km. In the case of the random walk (LSM-1) this trend also manifests in the early stage of the relative dispersion (Fig. 4.2, lower panel), and only a small parameter of $\sigma=125$ m/2 hours (corresponding to 1.7 cm/sec std of velocity fluctuations) can match the relative dispersion of HYCOM 1/100°. The local regime is slightly less pronounced with the random flight (LSM-2) and the averaged exponential regime of $D^2(t)$ in the first two days can be reproduced with $(\sigma, \tau)$ parameters resulting in comparatively higher diffusivities (Fig. 4.3 lower panel). The optimal parameters are summarized in Table 4.1 below:
Figure 4.2: $\lambda(\delta)$ (upper panel) and $D^2(t)$ (lower panel) of the LSM-1 (Random Walk) parameterization for three random-walk parameters corresponding to random kicks of 500 m, 250 m, and 125 m per 2-hour advection time-step.
Figure 4.3: $\lambda(\delta)$ (upper panel) and $D^2(t)$ (lower panel) of the LSM-2 (Random Flight) parameterization. Statistical parameters considered are velocity fluctuation stds $\sigma=0.5, 1.5$ cm/s and decorrelation time-scales $\tau=6\text{hr}, 1\text{day}$. 
The third LSM has the advantage of dramatically enhancing the relative dispersion while preserving to a certain extent the mesoscale transport barriers, and is best implemented for this reason in the GoM interior. However, it is less likely to perform on the shelf where the main constraints are a low relative dispersion and surface winds controlling the direction of the flow. A sensitivity study was conducted and the optimal parameter sets are shown in Fig. 4.4, corresponding to a very light Gaussian filter with a width of 1.5 grid-spacing for flow-decomposition, and doubling of the model velocity fluctuations, which amount to only about 2 cm/s. The decorrelation time scales on the shelf with this low-pass filter remain at 1 to 1.5 days. Doubling the decorrelation time-scales or leaving them unchanged lead to the best $\lambda(\delta)$ and $D^2(t)$ estimates so far compared to HYCOM 1/100°. The relative dispersion metrics are nonetheless insensitive to the LSM performance with regard to the wind constraint and the extent to which they can impact the transport pathways. A qualitative comparison is made in Fig. 4.5 with three relevant parameter sets. The added diffusion in the cases of the LSM-1,2 are minimum and applied independently of the underlying flow field. Provided the turbulent velocities are small enough compared to the model velocities, the direction of the Lagrangian transport should remain the same, and this is illustrated in Fig. 4.5 (upper panel) with a light random flight. On the other hand, the LSM-3 tends to amplify the turbulent Lagrangian accelerations where they are already predominant, and the impact on the direction of transport is prolonged by increasing the decorrelation time-scales. The middle and lower panels of Fig. 4.5 display the two LSM-3 tests previously discussed. The $(G1.5-2\sigma_m,2\tau_m)$-set is seen to modify substantially the direction of the flow. On the other hand, the $(G1.5-2\sigma_m)$-set mostly preserves the transport pathways of the model. Thus, the $(G1.5-2\sigma_m,2\tau_m)$-set is unsuitable for parameterization, in spite of its good relative dispersion statistics.

In summary, the addition of turbulence by the LSM is very limited on the shelf, and the LSM-1 and LSM-2 perform reasonably well. The LSM-3 can also be applied, but requires caution, especially in case of extreme wind events when the parameterization might overshoot the Lagrangian accelerations.

<table>
<thead>
<tr>
<th>LSM</th>
<th>$L_K$</th>
<th>$D_t$</th>
<th>$\sigma$ (cm/s)</th>
<th>$T^d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>LSM-1</td>
<td>125 m</td>
<td>2 hr</td>
<td>1.7</td>
<td>n/a</td>
</tr>
<tr>
<td>LSM-2</td>
<td>n/a</td>
<td>2 hr</td>
<td>0.5</td>
<td>1 day</td>
</tr>
<tr>
<td>LSM-2</td>
<td>n/a</td>
<td>2 hr</td>
<td>1.5</td>
<td>6 hr</td>
</tr>
<tr>
<td>LSM-2</td>
<td>n/a</td>
<td>2 hr</td>
<td>1.5</td>
<td>1 day</td>
</tr>
</tbody>
</table>

$^1L_K =$ displacement std  
$^2\sigma =$ velocity fluctuation std  
$^3D_t =$ advection time-step  
$^4\tau =$ decorrelation time-scale.
Figure 4.4: $\lambda(\delta)$ (upper panel) and $D^2(t)$ (lower panel) for the LSM-3 parameterization, with a flow decomposition obtained from the 1.5 grid strength Gaussian filter, and twice the model velocity fluctuations with twice or same decorrelation time scale. $\sigma_m \approx 2$ cm/s and $\tau_m = 1.5$ day.
Figure 4.5: Parameterized trajectories (green) superimposed to their model counterpart (black) for three relevant cases: an LSM-2 (upper panel) parameter set, and the two LSM-3 parameter sets of Fig. 4.4, illustrating the deviations of the trajectories from the wind constraint.

4.2 Tracer dispersion on the West Florida Shelf

The impact of the LSM-2 and LSM-3 on a tracer behavior is also studied by repeating the experiment on the GoM interior. The same patch of more than 100,000 particles was released on the northern part of the West Florida Shelf (Fig. 4.6) and advected monthly for a duration of 1 month.
The evolution of the tracers for the two models is displayed in the top panels of Fig. 4.7. We notice that there are some marked differences, mostly in the convoluted patterns of the tracers. Their general individual evolutions remain similar, since the surface circulation on the WFS is strongly influenced by the wind-forcing that is common to both simulations.

We then looked at the parameterized cases: two sets were implemented with the LSM-3 with G15-(1.5σ, 1.5τ) and G15-(2σ, 1τ) for the shelf (both satisfying the requirements of the previous section). G03-(2σ, 1τ) was used on the GoM-interior, and the results are plotted in the middle panel of Fig. 4.7. The other two sets considered were applied the LSM-2 parameterization with the reasonable parameters of (σ=0.7 cm/s τ=1 day) and (σ=1.5 cm/s, τ=1 day) and plotted in the bottom panel of Fig. 4.7. As expected from the previous results, the LSM-3 tended to enhance the dispersion by stretching the tracer into a filament-type of pattern. Doubling the velocity fluctuations while fixing the decorrelation time-scales produced more wiggles. The particles tended to stay confined to the WFS and the overall coverage was increased and resembled the tracer distribution of the reference run.

The impact of the LSM-2 on the tracer dispersion was very different. The addition of the random velocity fluctuations, albeit with a memory term was seen to diffuse the particle patch uniformly and to smear the filaments (cf Eq. 9). This effect was more pronounced with a higher-velocity fluctuation amplitude. The LSM-2 nevertheless succeeded at increasing the spatial coverage of the tracer, hence at improving the Lagrangian predictive capability of the numerical model.
Figure 4.7: Patch position after 1 month plotted with a different color for each launch/realization date in the reference run (top left), the model (top right) and different parameterized cases: The LSM-3 cases of (G15-1.5σ, 1.5τ) and (G15-2σ, 1τ) are in the middle left and right panels. The LSM-2 cases (0.7 cm/s, 1day) and (1.5 cm/s, 1day) are in the bottom panels. Color corresponds to days in the year 2010: days 61 (black), 91 (green), 121 (red), 151 (blue), 181 (magenta), 211 (cyan), 241 (yellow), and 271 (brown).
Below are the links to animations of section 4.2 experiments. Particle-based tracers are released on the West Florida shelf at the WFS location (cf Fig 4.6) and advected for a month by either the model (HYCOM 1/25°), the reference run (HYCOM1/100°), or the model combined with the LSM-3 applied to the shelf with the G15-\((1.5\sigma, 1.5\tau)\) parameter set. The last set of animations shows the evolution of the tracers from model, LSM and reference run, since the wind control on the shelf allows (to a certain extent) for a more direct qualitative comparison. These animations are also included in the DVD attached to this report under the directory *animBOEM*.

WFS HYCOM1/25° patch launches 2 through 5:
https://dl.dropboxusercontent.com/u/21001310/animsBOEM/modWFS-02.gif
https://dl.dropboxusercontent.com/u/21001310/animsBOEM/modWFS-03.gif
https://dl.dropboxusercontent.com/u/21001310/animsBOEM/modWFS-04.gif
https://dl.dropboxusercontent.com/u/21001310/animsBOEM/modWFS-05.gif

WFS HYCOM1/100° patch launches 2 through 5:
https://dl.dropboxusercontent.com/u/21001310/animsBOEM/1kmWFS-02.gif
https://dl.dropboxusercontent.com/u/21001310/animsBOEM/1kmWFS-03.gif
https://dl.dropboxusercontent.com/u/21001310/animsBOEM/1kmWFS-04.gif
https://dl.dropboxusercontent.com/u/21001310/animsBOEM/1kmWFS-05.gif

WFS comparison model (blue), reference (green), LSM-3 (red) launches 2 through 5:
https://dl.dropboxusercontent.com/u/21001310/animsBOEM/WFScomp-02.gif
https://dl.dropboxusercontent.com/u/21001310/animsBOEM/WFScomp-03.gif
https://dl.dropboxusercontent.com/u/21001310/animsBOEM/WFScomp-04.gif
https://dl.dropboxusercontent.com/u/21001310/animsBOEM/WFScomp-05.gif
5 Summary

Recent observations and high-resolution simulations from state-of-the art ocean models have indicated that SMS motions are present in the GoM. They tend in part to emerge from mixed layer instabilities when weak stratification and mixed layer depth permit vertical recirculations along the edges of front. SMS filaments and eddies act to enhance particle pair separations, hence dispersion and mixing at their own scales. This translates into an increase in the scale-dependent relative dispersion (or FSLE) at scales up to the Rd, slightly modifying the mesoscale transport barriers, and causing particles to exit and join other mesoscale transport pathways. OGCMs used at the present time for ocean forecasts and Lagrangian prediction problems have mesh sizes around 1/25°, which does not allow them to resolve small-scale motions. This results in an underestimated strain field and low relative dispersion at the SMS.

A novel method was applied to the GoM to correct the surface Lagrangian transport by combining LSMs in the SMS range, with LCS from the ocean model in the mesoscale range. More specifically, the objective has been to enhance the scale-dependent relative dispersion of a HYCOM 1/25° simulation from the small scales O(1 km) to twice the Rd, while preserving the relative dispersion at the larger scales, to maintain the deterministic LCS.

We have relied mostly on a higher (1/100°) resolution HYCOM simulation of the GoM to test the performance of the LSMs. This numerical model captures SMS features in the 3 to 10 km range, yielding FSLE values at the SMS up to twice those of HYCOM 1/25°. Multi-scale dispersion experiments from the GLAD drifter data around De Soto Canyon indicated a local dispersion regime with even higher FSLE estimates in the SMS range up to the Rd, and confirming the strong influence of SMS features on dispersion and mixing at those scales. The GLAD FSLE was recomputed after low-pass filtering the drifter trajectories to remove diurnal oscillations and to obtain FSLE estimates more consistent with cumulative dispersion for LSM implementation with velocity outputs availability of about 1 day. This resulted in substantially lower FSLE values, more similar to those of HYCOM 1/100° at scales above 5 km, thus matching the intermediate scale FSLE of the high-resolution model. The HYCOM 1/100° synthetic trajectories can therefore be considered a valid reference for the LSM adaptation.

In the GoM interior, the LSM-3 was applied. The flow-decomposition was chosen to preserve most of the mesoscale pathways from the turbulent velocity fluctuations enhanced by the missing component \( \eta(t) \). These fluctuations in turn enhanced the scale-dependent rates of dispersion at \( \delta \leq 2 \) Rd, and resulted in a significant improvement of the relative dispersion to levels close to HYCOM 1/100°. Evolution of tracer dispersions illustrated the drastic differences between this stochastic model and the classic LSM-1 and LSM-2 (Random Walk and Random Flight, respectively), as it preserves the tendency of the flow to stretch and elongate patches of particles. This is due to the fact that the addition of diffusion by \( \eta \) is continuous in time, and to a certain degree correlated in space. The increase in the Lagrangian velocity fluctuations is more pronounced whenever particles get advected by highly kinetic mesoscale features, thus resulting in a spatially polarized parameterization where the edges of the mesoscale features are favored. The modified fluctuations are still an order of magnitude lower than the typical velocity scales in
the region, so that the corrected Lagrangian transport is still constrained by the mesoscale transport barriers. A range of optimal parameters is given in Table 3.1 for the HYCOM1/25° simulation (with a horizontal resolution of 3 to 4 km used nowadays by state-of-the-art ocean prediction models). These include choices of filters for an adequate flow-decomposition and corresponding Markov-1 statistical parameters.

The LSM-3 was also applied to the geostrophic velocities of the AVISO altimetry product during the GLAD experiment, since it constituted the closest estimate of the true mesoscale field. Altimetry products usually have a coarse resolution and weaker kinetic energy due to spatio-temporal interpolations compensating for satellite track irregularity, resulting in a very low strain-field. Strong filters are thus necessary to obtain non-negligible turbulent velocities in order to implement the LSM-3. The temporal moving averages of 11 and 21 days were used for flow-decomposition and a doubling of the Markov 1 parameters enhanced substantially the Lyapunov exponent. Tracer releases displayed significant qualitative improvement in particle dispersion, while both GLAD drifters and parameterized particles kept following the mesoscale pathways of AVISO. The optimal statistical parameters are given in Table 3.2.

On the continental shelf, either one of the 3 LSMS can be applied: The surface circulation on the shelf as resolved by the numerical models misses physics such as tides in this case, non-hydrostatic effects, mixed-layer dynamics, etc. It is found nevertheless, that the wind-forcing exerts a strong constraint on the flow direction, affecting the horizontal velocities in a synoptic pattern with weak dependence on the particle location. It is also characterized by weak relative dispersion (compared to the GoM interior), which favors the application of the LSM-1 and LSM-2. The corresponding parameters are given in Table 4.1. The LSM-3 implementation is also possible, albeit with a weak filter, so that changes in the flow direction remain minimal, compared to the changes imposed by the surface winds.

To conclude, the innovative aspect of this work has been to rely on the scale-dependent relative dispersion to implement recent and classic statistical Lagrangian subgridscale models (or LSM) in an eddy-permitting ocean model with a grid resolution of 1/25°, consistent with today’s ocean forecast models used in the GoM. The performance of the LSMS was evaluated based on an SMS-permitting ocean model of 1 km resolution. A newly developed LSM based on the Markov 1 approximation (referred here as the LSM-3) was adapted to both the GoM interior and with some caution to the continental shelf. The LSM-3 successfully compensated for the unrealistically low strain levels of a HYCOM 1/25° simulation at the SMS, while preserving the large-scale relative dispersion of the model. Unlike other stochastic models, the LSM-3 preserves the tracer filamentation, due in part to its type of flow-decomposition. The classic Random Walk and Random Flight models were found to be suitable to the continental shelf, where the surface flow is constrained by lower kinetic energy levels and by the surface winds. The impact of LSM-3 on particle trajectories was evidenced by the substantial improvement in particle-based tracer distributions, which were seen to approach statistically the spatial coverage of the SMS-permitting ocean model. For today’s ocean forecast models, benefitting from a more accurate representation of the mesoscale field, yet limited in their capabilities to capture SMS motions, the LSM implemented in this study for the surface circulation of the GoM will lead to a significant improvement in oil spill risk analysis.
References


Iskandarani, M., and Mensa, J. 2013, oral communication, unpublished data.


The Department of the Interior Mission

As the Nation’s principal conservation agency, the Department of the Interior has responsibility for most of our nationally owned public lands and natural resources. This includes fostering sound use of our land and water resources; protecting our fish, wildlife, and biological diversity; preserving the environmental and cultural values of our national parks and historical places; and providing for the enjoyment of life through outdoor recreation. The Department assesses our energy and mineral resources and works to ensure that their development is in the best interests of all our people by encouraging stewardship and citizen participation in their care. The Department also has a major responsibility for American Indian reservation communities and for people who live in island territories under US administration.

The Bureau of Ocean Energy Management

As a bureau of the Department of the Interior, the Bureau of Ocean Energy (BOEM) primary responsibilities are to manage the mineral resources located on the Nation’s Outer Continental Shelf (OCS) in an environmentally sound and safe manner.

The BOEM Environmental Studies Program

The mission of the Environmental Studies Program (ESP) is to provide the information needed to predict, assess, and manage impacts from offshore energy and marine mineral exploration, development, and production activities on human, marine, and coastal environments.