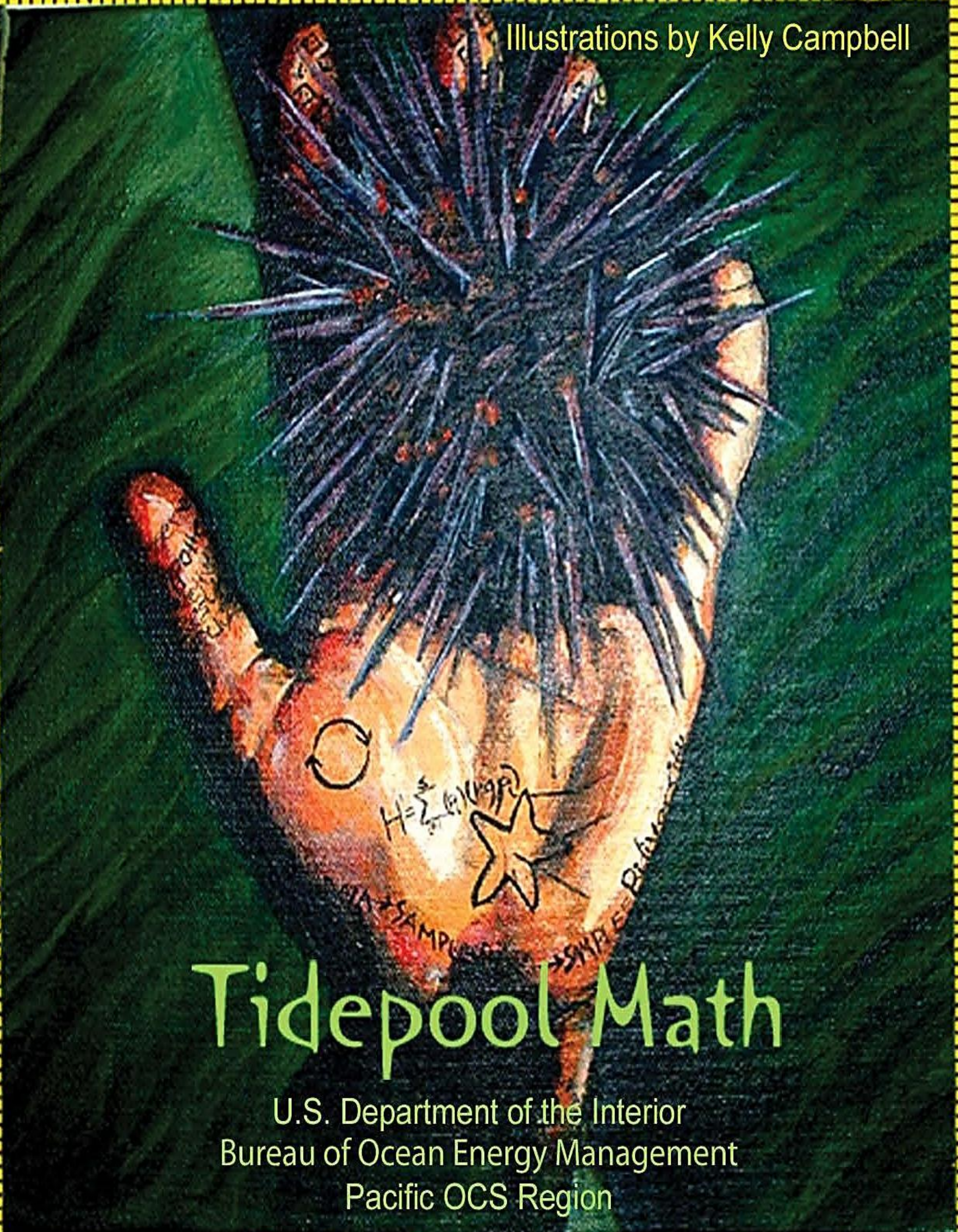
Illustrations by Kelly Campbell



Tidepool Math

U.S. Department of the Interior
Bureau of Ocean Energy Management
Pacific OCS Region

Tidepool Math
High school

Process Skills Used in this Lesson

1. Be able to set up a random, systematic, and targeted sampling approach for a fixed plot of plants or animals.
2. Be able to determine whether a count of individuals or a percent cover measurement is best to determine abundance for a given plot.
3. Be able to compare two or more locations and determine if the living communities are statistically different from each other.

Learning Objectives

- Increased awareness of the diversity of animals and plants in rocky intertidal habitats.
- Understand the differences between random, systematic and targeted sampling approaches
- Compare estimates, counts, and means. Discuss outliers.
- Understand simple statistical concepts and tools that are used to design a study and analyze environmental data.

Background for Teachers

Proper adherence to the Scientific Method involves defining the question to be studied and setting up a study design which will answer the question being asked. Fundamental to applying scientific method is the student's ability to define hypotheses about the phenomenon, system or process being studied. The null hypothesis (e.g., there is no statistical difference between two populations) is central to proper scientific study as well as the basis for statistical analytic models.

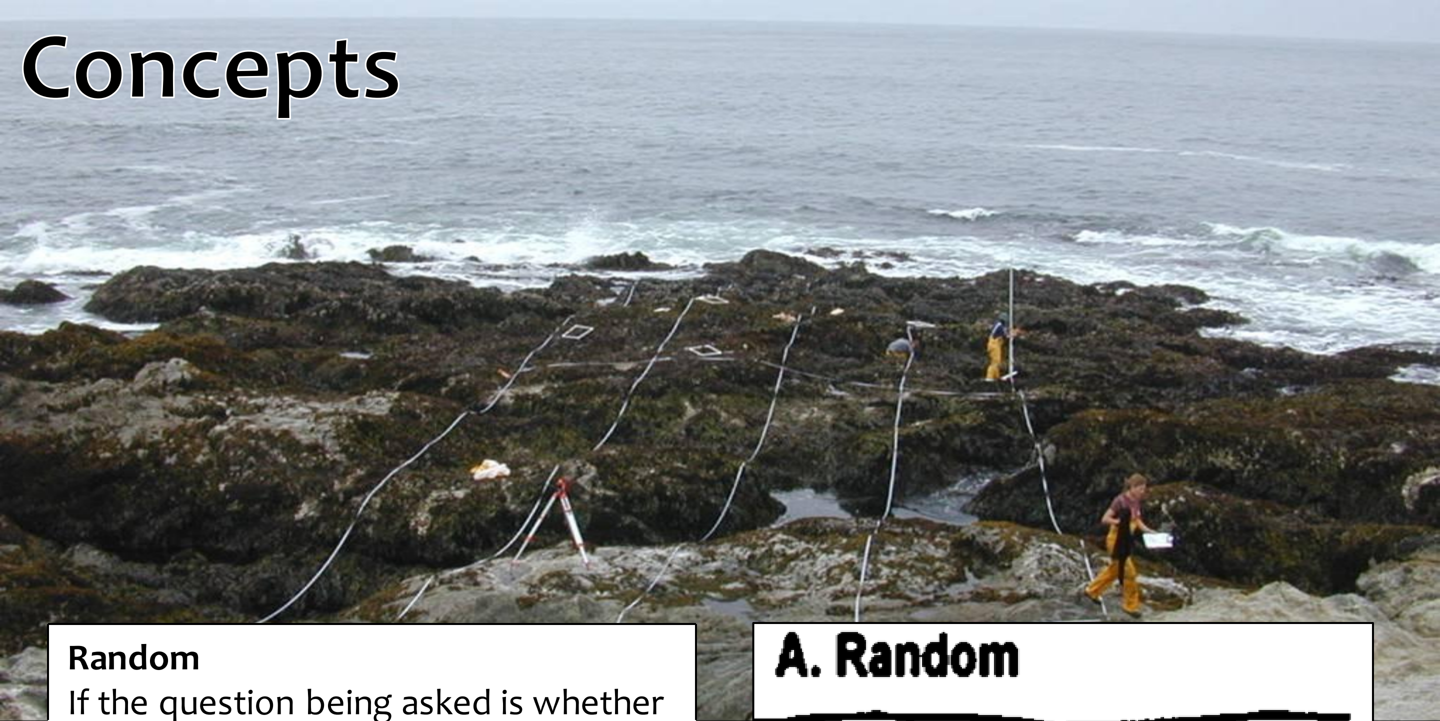
In this first exercise, students will be shown how the study design, whether random, systematic or targeted, will answer different questions and have different advantages and disadvantages. This basic concept can be visually demonstrated using a standard rocky intertidal plot sampling design.

In the second and third exercises, the student is challenged to pose a hypothesis about the distribution of plants or animals that live in the rocky intertidal coastal environment. Students are shown the application of statistics and statistical analysis to contemporary problems in marine ecology.

Abundance of a given population is determined either through counts or measuring percent cover. In a rocky intertidal habitat, both are used. Percent cover is used for algae, densely populated animal communities such as barnacles, and colonial animals such as small anemones. Counts of individuals are used to determine the abundance of different invertebrates like snails, and sea stars. Depending on the study and the scale of the effort, mussels may be sampled either with counts or cover.

Biologists with several agencies and universities currently monitor rocky intertidal habitats along the California coast. This group, MARINe, or the Multi-Agency Rocky Intertidal Network, has established 55 permanent sites where they sample every fall and spring. Species photographed in fixed plots include mussels, anemones, acorn barnacles, gooseneck barnacles, turf algae, rockweed, and red fleshy algae. Notes about the plots are taken by biologists at the site but the " counts" of the species in the photos are done in the laboratory. Each photo is displayed on a computer screen and overlaid with 100 points. Species under each point are counted to determine the percent cover of each species identified. Species counted and measured as individuals in fixed plots include sea stars, owl limpets, and black abalone. Several varieties of small invertabrates such as snails and chitons are also counted and measured in the photoplots by biologists at the site.

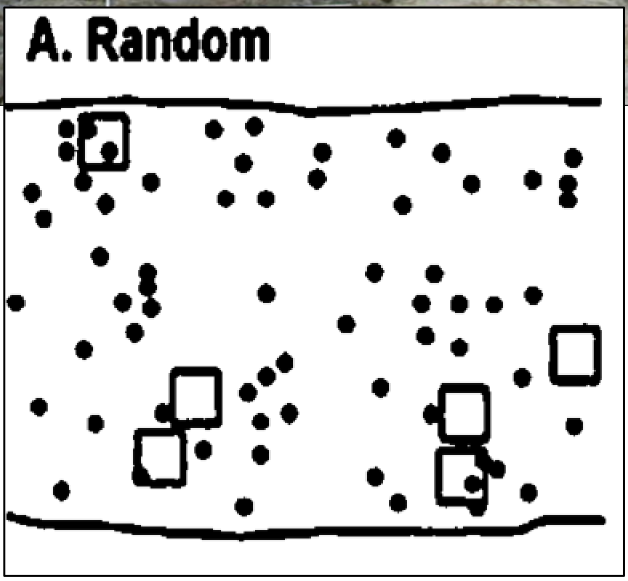
Concepts



Random

If the question being asked is whether populations (numbers or density of animals) differ at different locations, the focus of the sampling design is to obtain representative samples of the populations being compared. The best statistical design for this questions is random sampling within each population (or plot). This reduces the bias that the scientist might make in assigning sampling locations within a plot to denser or sparser areas, unintentionally avoiding edges of the study area, unintentionally aggregating samples etc. A disadvantage of random sampling is that a large number of samples may be required for good statistical power in populations that are patchy or unevenly distributed.

Random number tables can be used to select numbers which can be used, for example, as coordinates for transect lines.



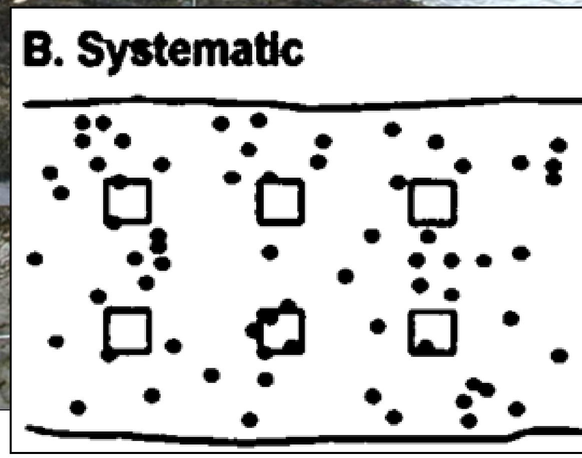
Random sampling is not always feasible, either due to time or physical constraints. Completely random sampling may not be appropriate if populations are obviously distributed in a systematic fashion or if the question being asked is about particular habitats and not the general community. In those latter cases, Systematic (Also known as Stratified Random) sampling is a better choice. Other approaches commonly used include systematic sampling and targeted sampling.

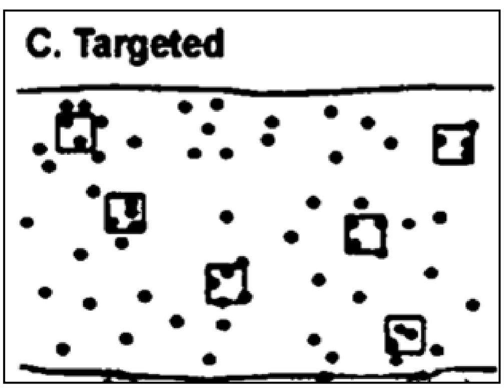
Concepts Con’t

Systematic

A systematic sampling design places sampling units uniformly through the study site. Transect lines spaced at regular intervals is an example of a systematic approach. The advantages are that it can be much easier in practice to do, and the estimate of the mean may be more accurate that with random samples.

The main disadvantage is that it is not random, and therefore will not meet certain statistical test criteria.





Targeted

The rocky intertidal sampling chosen for the MARINe core monitoring program is a targeted design. This design focuses the time spent by researchers on sampling targeted populations where the abundance of a given species of interest is highest. For example, photo plots are targeted in low intertidal mussel beds for mussels, and the high intertidal for barnacles.

This design is appropriate when certain species are being targeted for study. It is an unacceptable design when the questions asked is “what is the average density of a population in a habitat?”

This is a useful design in many biological habitats because species are often clustered by physical parameters such as tidal height, distance form shore, water depth, distance from a stream, elevation, etc.

To reduce bias in the sampling, it is best if individual sampling units are randomly placed within a targeted population. For example, in the MARINe programs, five photo quadrats were randomly placed within each defined target population.

EXERCISE 1: Sampling a tidepool plot

Learning Objectives	Process Skills
Determine the abundance of a plot by comparing estimates, counts, means, and percent cover	Determine whether a count of individuals or a percent cover measurement is best used when evaluating abundance.

Introduction:

Discuss the term *abundance* and how one might measure abundance.
Discuss the concept of *species richness* and *biodiversity* (**see glossary).**

- Biodiversity is an important concept in ecology. It relates to species richness and the relative distribution of the numbers of individuals among all the species present in the area. It is a term students will hear about and read about in contemporary articles about the earth's environment. In general, higher biodiversity is characteristic of more mature and less disturbed habitats.

Note that biodiversity is not synonymous with species richness although habitats with high biodiversity tend to have many more species that habitats with low biodiversity.

The easiest way to measure the abundance of *discrete individuals* is to count them. However, if the item being counted is a colonial animal or plant, it is very hard to count them. A measure of *percent cover,* or amount of area covered by that species, is used to determine abundance.

Look at the photos provided. In the photo of mussels, these individuals can be counted. In the field, scientists both measure and count individual animals such as mussels, limpets, sea stars and abalone.

In the photo of anemones, this colonial form grows so close together and has attached so many shell particles that it is almost possible to identify individual anemones. The only appropriate way to measure abundance is by looking at percent cover. If the anemones covered the entire plot, it would be recorded as 100% cover.

Classroom Activity 1: Counting Mussels

Materials: Enlarged photo of mussel plot (2’ by 3’), sheet of laminate, dry erase markers

Preparation: enlarge mussel plot photo by 2’ by 3’ and divide into quarters with masking tape

a. Have the students estimate the number of mussels in the first plot.
b. Lay the photo on the floor and cover with laminate.
c. Ask four students to count the mussels in each quadrant and add the values of the four quadrants to get a total for the entire plot.
d. Compare the total number estimated for the plot to the values obtained by taking a value for any one quadrant and multiplying by 4. Determine the mean value of mussels for each quadrant. Discuss any *outliers.*

Classroom Activity 2: Estimating Percent Cover

Materials: Enlarged photo of anemone (2’ by 3’), sheet of laminate, dry erase markers

Preparation: Enlarge anemone plot to 2’ by 3’ and score 2’ by 3’ sheet of laminate with 100 evenly spaced points

a. Looking at the photo, ask each student to estimate the percent cover of anemones and green algae (Ulva).
b. Overlay laminate with 100 evenly spaced points. Ask several students to identify what is under each point 1) anemone, 2) green algae or 3) other (mussel, rock, sand, etc.)
c. Calculate percentages for “1”,”2”, and “3”, or percent cover (for example: # of “1’s” / 100).
d. Compare calculated percentages with the original estimates.

Exercise 2: Methodology: Random, Systematic, and Targeted sampling.

Learning Objectives	Process Skills
Take a sample of a mussel plot using random, systematic, and targeted sampling approaches.	Compare three sampling schemes; random, systematic, and targeted.

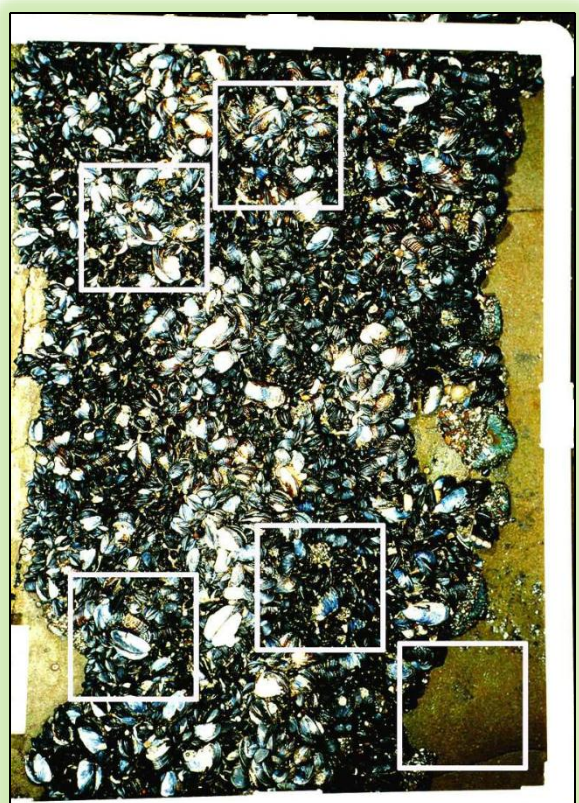
Note: In order to demonstrate this concept in the classroom setting, we are constructing a hypothetical sampling within the mussel photo, assuming that the 2' by 3' plot is representative of the entire habitat site. Realistically this would be done on a much larger scale. Explain to the class that normally this type of sampling would be done with a plot encompassing an entire habitat.

Classroom Activity 3:

Materials: Enlarged mussel photo, dry erase markers, 5 small laminate 2" squares

Preparation: cut sheet of laminate into five 6" squares, (note: the size of the square will change according to the size of enlarged photo) Enlarge mussel plot photo to 2' by 3' (if not done with previous exercise)

a. Place enlarged 2'x3' mussel photo on the floor.
b. Place measuring tapes or yard sticks along the x and y axis of your photo.
c. Use a random number table to generate 5 numbers between 1 and 25 and 5 numbers between 1 and 36. Pair numbers to generate 5 coordinates which can be marked on the laminate.
d. Make 5 sampling "squares" by cutting laminate into 2"squares.
e. Lay the 5 squares onto the photo using the paired coordinates to locate the upper left corner of each square. Count mussels under each square. Add numbers and calculate a mean.

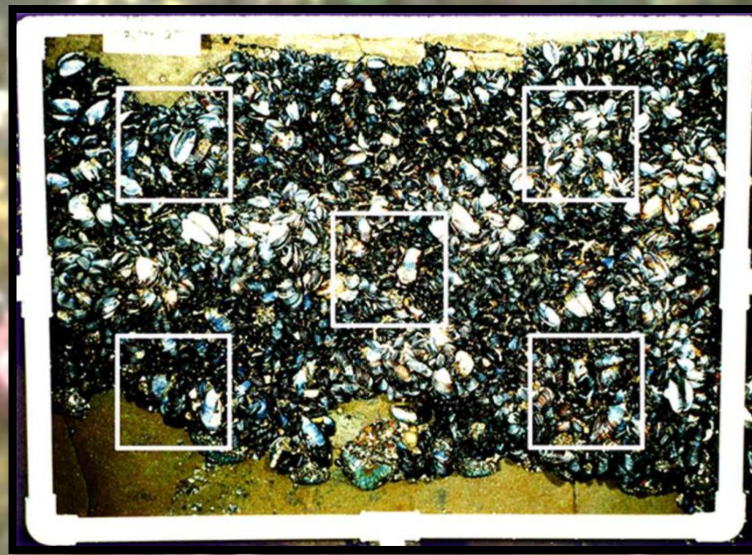


CLASSROOM ACTIVITY 4: Sample the plot systematically.

Preparation: cut sheet of laminate into five 6” squares, Enlarge mussel plot photo to 2’ by 3’ (if not done with previous exercise)

Materials: Enlarged mussel photo, dry-erase markers, 5 laminate 6” squares.

a. **Place the five squares at regular intervals on the plot like a “five” on a dice. Measure distance between the squares so that they are the same distances apart.**
b. **Record the mussels counted under each square. Calculate the mean.**

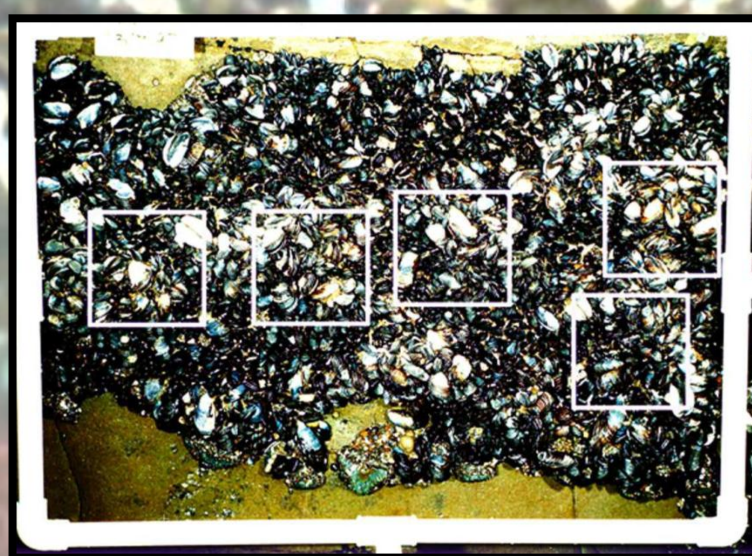


CLASSROOM ACTIVITY 5: Sample the plot with a targeted scheme.

Preparation: cut sheet of laminate into five 6” squares, Enlarge mussel plot photo to 2’ by 3’ (if not done with previous exercise)

Materials: Enlarges mussel photo, dry-erase marker, 5 small laminate squares

a. **Determine where the mussels are concentrated on the photo. Generate random number pairs which only fall within this area.**
b. **Place the squares randomly within the targeted area.**
c. **Record the mussels counted under each square. Calculate the mean.**



Compare the three sampling schemes with the original counts of mussels in the plot. Determine which sampling method best describes the number of mussels in the plot.

Exercise 3: Species Diversity Index

Learning Objectives	Process Skills
Compare the species richness and species diversity at two intertidal site (photoquadrats).	Calculate a species diversity index (a mathematical index or measure of diversity).

Introduction:

The Shannon-Weiner Diversity Index is a simple index that is used frequently in ecological analyses to calculate *biodiversity.* **It is calculated by the following formula:**

$$H' = \sum_{i=1}^{s} (pi)(\log pi)$$

Where s is *species richness* and pi is the proportion of each organism in the photoquadrat.

Species richness is simply the number of different species counted in all samples for each photoquadrat.

Classroom Activity 6: Calculating Biodiversity

Materials: Two large laminated photoquadrats (separate locations), small 6" sample squares, data recording sheets, random number table.

1. **Sample the mussel plot randomly.**
   a. Place enlarged 2'x3' mussel photo on the floor.
   b. Place measuring tapes or yardsticks along the x and y axis of your photo.
   c. Use a random number table to generate 5 numbers between 1 and 24 and 5 numbers between 1 and 36. Pair numbers to generate 5 coordinates which can be marked on the laminate.
   d. Make 5 sampling "squares" by cutting laminate into 6" squares and covering edges with masking tape.
   e. Lay the five squares onto the photo using the paired coordinates to locate the upper left corner of the square. Count the number or percent cover of each separate species under each square and record on data sheet.

2. Calculate the Shannon-Weiner Species Diversity Index for each photoquadrat.

3. Compare a) the species richness and; b) Shannon-Weiner Diversity Index of the two photoquadrats.

Exercise 4: Statistical Analysis of Photoquadrats

Learning Objectives	Process Skills
Compare mean density of mussels (or other abundant species) from two locations	Determine if there is a statistically significant difference in mean density of mussels in different locations

Introduction:

This exercise compares the species richness and species diversity at two intertidal sites (photoquadrats). The mean densities of populations of mussels (students can select other abundant species as options) from two locations (photoquadrats) will be compared. Students will determine if there is a statistically significant difference in the mean (average) *density* **(numbers) of mussels.**

Hypothesis: There is no difference between the two locations' (photoquadrats) average number of mussels (or other organism selected).

Materials: Two large laminated photoquadrats (separate locations), small 2" sample squares, data recording sheets, random number table, Analysis of Variance (ANOVA) worksheet, F statistic Table.

1. **Sample the mussel plot randomly**

a. Place enlarged 2'x3' mussel photo on the floor.

b. Place measuring tapes or yardsticks along the x and y axis of your photo.

c. Use a random number table to generate 5 numbers between 1 and 24 and 5 numbers between 1 and 36. Pair numbers to generate 5 coordinates which can be marked on the laminate.

d. Make 5 sampling "squares" by cutting laminate into 2" squares and covering edges with masking tape.

e. Lay the five squares onto the photo using the paired coordinates to locate the upper left corner of the square. Count mussels under each square and record on data sheet.

2. Using the ANOVA worksheet, calculate the means and variances (total, within, and between), and the F statistic. Using the F table, look up whether the F statistic is significant.

3. Compare the original null hypothesis (there is no statistical difference between the two populations) with the result from the F table.

Glossary

Abundance: a measure of the size of the population. Abundance is measured by counting individuals (density) per unit area (square meter), or by determining the amount of area covered (percent cover). Density is the most common measure of abundance for animals such as sea stars and abalone since individuals are widely spaced and easily counted. For animals such as colonial anemones and for many plants such as sea lettuce, rockweed and coralline algae, percent cover is better measure of abundance.

Species Richness: the total number of different species in a community. Undisturbed communities in habitats such as mussel beds will have a large number of different kinds of plants and animals species. The way to determine species richness is to take a complete inventory of all the different species in an area.

Biodiversity: the total number of different species in a community. Undisturbed communities in habitats such as mussel beds will have a large number of different kinds of plants and animal species.

Percent Cover: measure of abundance found by determining the amount of area being occupied by a given species.

Density: measure of abundance found by counting individuals in a unit area (e.g. square meter).

Discrete individuals: individual animals or plants that are widely enough spaced or distinctive enough to be counted.

Outliers: a term referring to data which does not appear to group with the majority of the data. For example, in a list of numbers—1,2,4,2,6,3,56—"56" is an outlier. What you do with that outlier requires critical thinking. An outlier may be a data point recorded incorrectly. (e.g., typographic error) or it may represent a completely valid point. For example, if plotting population data locates a gray whale in Oklahoma City, it can be assumed to be an erroneous point and should be tossed. However, If you want to estimate the size of a population and counts of sample plots are: 0, 113, 114, 117, 124, 128, 130, 500– neither of the outliers (in this case, "0" and "500") should be thrown out without checking back with the person recording the data because watch number may be a valid count and is needed to accurately estimate the total population.

Science in the Tidepool

Analysis of Variance (ANOVA)

Student Worksheet No. 1

Photoquadrat $_A$ (Site) No. 1	Photoquadrat $_B$ (Site) No. 2	Photoquadrat $_C$ (Site) No. 3	
Sample 1 = X_A X_A^2=	Sample 1 = X_B X_B^2=	Sample 1 = X_C X_C^2=	
Sample 2 = X_A X_A^2=	Sample 2 = X_B X_B^2=	Sample 2 = X_C X_C^2=	
Sample 3 = X_A X_A^2=	Sample 3 = X_B X_B^2=	Sample 3 = X_C X_C^2=	
Sample 4 = X_A X_A^2=	Sample 4 = X_B X_B^2=	Sample 4 = X_C X_C^2=	
Sample 5 = X_A X_A^2=	Sample 5 = X_B X_B^2=	Sample 5 = X_C X_C^2=	
$\sum X_A =$	$\sum X_b =$	$\sum X_c =$	$\sum X = \sum X_A + \sum X_b + \sum X_c$
$\sum X_A^2 =$	$\sum X_B^2 =$	$\sum X_C^2 =$	$\sum X^2 = \sum X_A^2 + \sum X_B^2 + \sum X_C^2$
N_A= 5	N_B= 5	N_C= 5	N (total number of samples) = $N_A + N_B + N_C$

X is the number of mussels (or other organisms) counted.

Science in the Tidepool

Analysis of Variance (ANOVA)
Student Worksheet No. 2

Step 1. Calculate Variances (Sum of Squares, SS) from ANOVA Worksheet No. 1

$$SS_{Total} = \sum X^2 - \frac{(\sum X)^2}{N}$$

$$SS_{Between} = \frac{(\sum X_A)^2}{N_A} + \frac{(\sum X_B)^2}{N_B} + \frac{(\sum X_B)^2}{N_C} + \ldots$$

$$SS_{Within} = SS_{Total} - SS_{Between}$$

Step 2. Complete the Anova Table

Variance Source	Degrees of Freedom	Sum of Squares (SS)	Mean Square
Between	1 (for 2 photoquadrats)	$SS_{Between}$	$\frac{SS_{Between}}{1} =$
Within	8	SS_{Within}	$\frac{SS_{Within}}{8} =$
Total	N – 1 = 9	SS_{Total}	

Step 3. Calculate the F Statistic

$$F_{1,8} = \frac{MeanSquare_{Between}}{MeanSquare_{Within}} =$$

Where $F_{1,8}$ subscripts indicate the degrees of freedom for this analysis.

Step 4. Look up F in Table.

A-1 RANDOM NUMBERS

10	09	73	25	33	76	52	01	35	86	34	67	35	48	76	80	95	90	91	17	39	29	27	49	45
37	54	20	48	05	64	89	47	42	96	24	80	52	40	37	20	63	61	04	02	00	82	29	16	65
08	42	26	89	53	19	64	50	93	03	23	20	90	25	60	15	95	33	47	64	35	08	03	36	06
99	01	90	25	29	09	37	67	07	15	38	31	13	11	65	88	67	67	43	97	04	43	62	76	59
12	80	79	99	70	80	15	73	61	47	64	03	23	66	53	98	95	11	68	77	12	17	17	68	33
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98	52	01	77	67	14	90	56	86	07	22	10	94	05	58	60	97	09	34	33	50	50	07	39	98
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83	45	29	96	34	06	28	89	80	83	13	74	67	00	78	18	47	54	06	10	68	71	17	78	17
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69	91	62	68	03	66	25	22	91	48	36	93	68	72	03	76	62	11	39	90	94	40	05	64	18
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91	49	91	45	23	68	47	92	76	86	46	16	28	35	54	94	75	08	99	23	37	08	92	00	48
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15	47	44	52	66	95	27	07	99	53	59	36	78	38	48	82	39	61	01	18	33	21	15	94	66
94	55	72	85	73	67	89	75	43	87	54	62	24	44	31	91	19	04	25	92	92	92	74	59	73
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11	66	44	98	83	52	07	98	48	27	59	38	17	15	39	09	97	33	34	40	88	46	12	33	56
48	32	47	79	28	31	24	96	47	10	02	29	53	68	70	32	30	75	75	46	15	02	00	99	94
69	07	49	41	38	87	63	79	19	76	35	58	40	44	01	10	51	82	16	15	01	84	87	69	38

A-7a F DISTRIBUTION, UPPER 5 PER CENT POINTS ($F_{.95}$)

DEGREES OF FREEDOM FOR NUMERATOR

DEGREES OF FREEDOM FOR DENOMINATOR	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	∞
1	161	200	216	225	230	234	237	239	241	242	244	246	248	249	250	251	252	253	254
2	18.5	19.0	19.2	19.2	19.3	19.3	19.4	19.4	19.4	19.4	19.4	19.4	19.4	19.5	19.5	19.5	19.5	19.5	19.5
3	10.1	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.70	8.66	8.64	8.62	8.59	8.57	8.55	8.53
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.86	5.80	5.77	5.75	5.72	5.69	5.66	5.63
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.62	4.56	4.53	4.50	4.46	4.43	4.40	4.37
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.94	3.87	3.84	3.81	3.77	3.74	3.70	3.67
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.51	3.44	3.41	3.38	3.34	3.30	3.27	3.23
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.22	3.15	3.12	3.08	3.04	3.01	2.97	2.93
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.01	2.94	2.90	2.86	2.83	2.79	2.75	2.71
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.85	2.77	2.74	2.70	2.66	2.62	2.58	2.54
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.79	2.72	2.65	2.61	2.57	2.53	2.49	2.45	2.40
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.62	2.54	2.51	2.47	2.43	2.38	2.34	2.30
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.60	2.53	2.46	2.42	2.38	2.34	2.30	2.25	2.21
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.53	2.46	2.39	2.35	2.31	2.27	2.22	2.18	2.13
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.48	2.40	2.33	2.29	2.25	2.20	2.16	2.11	2.07
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.42	2.35	2.28	2.24	2.19	2.15	2.11	2.06	2.01
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.38	2.31	2.23	2.19	2.15	2.10	2.06	2.01	1.96
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.34	2.27	2.19	2.15	2.11	2.06	2.02	1.97	1.92
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.31	2.23	2.16	2.11	2.07	2.03	1.98	1.93	1.88
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.28	2.20	2.12	2.08	2.04	1.99	1.95	1.90	1.84
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.25	2.18	2.10	2.05	2.01	1.96	1.92	1.87	1.81
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.23	2.15	2.07	2.03	1.98	1.94	1.89	1.84	1.78
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.20	2.13	2.05	2.01	1.96	1.91	1.86	1.81	1.76
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.18	2.11	2.03	1.98	1.94	1.89	1.84	1.79	1.73
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.16	2.09	2.01	1.96	1.92	1.87	1.82	1.77	1.71
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.09	2.01	1.93	1.89	1.84	1.79	1.74	1.68	1.62
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.00	1.92	1.84	1.79	1.74	1.69	1.64	1.58	1.51
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.92	1.84	1.75	1.70	1.65	1.59	1.53	1.47	1.39
120	3.92	3.07	2.68	2.45	2.29	2.18	2.09	2.02	1.96	1.91	1.83	1.75	1.66	1.61	1.55	1.50	1.43	1.35	1.25
∞	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83	1.75	1.67	1.57	1.52	1.46	1.39	1.32	1.22	1.00

Interpolation should be performed using reciprocals of the degrees of freedom.

By permission of Prof. E. S. Pearson from M. Merrington, C. M. Thompson, "Tables of percentage points of the inverted beta (F) distribution," *Biometrika*, vol. 33 (1943), p. 73.

DEGREES OF FREEDOM FOR NUMERATOR

DEGREES OF FREEDOM FOR DENOMINATOR	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	∞
1	4,052	5,000	5,403	5,625	5,764	5,859	5,928	5,982	6,023	6,056	6,106	6,157	6,209	6,235	6,261	6,287	6,313	6,339	6,366
2	98.5	99.0	99.2	99.2	99.3	99.3	99.4	99.4	99.4	99.4	99.4	99.4	99.4	99.5	99.5	99.5	99.5	99.5	99.5
3	34.1	30.8	29.5	28.7	28.2	27.9	27.7	27.5	27.3	27.2	27.1	26.9	26.7	26.6	26.5	26.4	26.3	26.2	26.1
4	21.2	18.0	16.7	16.0	15.5	15.2	15.0	14.8	14.7	14.5	14.4	14.2	14.0	13.9	13.8	13.7	13.7	13.6	13.5
5	16.3	13.3	12.1	11.4	11.0	10.7	10.5	10.3	10.2	10.1	9.89	9.72	9.55	9.47	9.38	9.29	9.20	9.11	9.02
6	13.7	10.9	9.78	9.15	8.75	8.47	8.26	8.10	7.98	7.87	7.72	7.56	7.40	7.31	7.23	7.14	7.06	6.97	6.88
7	12.2	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72	6.62	6.47	6.31	6.16	6.07	5.99	5.91	5.82	5.74	5.65
8	11.3	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91	5.81	5.67	5.52	5.36	5.28	5.20	5.12	5.03	4.95	4.86
9	10.6	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35	5.26	5.11	4.96	4.81	4.73	4.65	4.57	4.48	4.40	4.31
10	10.0	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94	4.85	4.71	4.56	4.41	4.33	4.25	4.17	4.08	4.00	3.91
11	9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.63	4.54	4.40	4.25	4.10	4.02	3.94	3.86	3.78	3.69	3.60
12	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39	4.30	4.16	4.01	3.86	3.78	3.70	3.62	3.54	3.45	3.36
13	9.07	6.70	5.74	5.21	4.86	4.62	4.44	4.30	4.19	4.10	3.96	3.82	3.66	3.59	3.51	3.43	3.34	3.25	3.17
14	8.86	6.51	5.56	5.04	4.70	4.46	4.28	4.14	4.03	3.94	3.80	3.66	3.51	3.43	3.35	3.27	3.18	3.09	3.00
15	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89	3.80	3.67	3.52	3.37	3.29	3.21	3.13	3.05	2.96	2.87
16	8.53	6.23	5.29	4.77	4.44	4.20	4.03	3.89	3.78	3.69	3.55	3.41	3.26	3.18	3.10	3.02	2.93	2.84	2.75
17	8.40	6.11	5.19	4.67	4.34	4.10	3.93	3.79	3.68	3.59	3.46	3.31	3.16	3.08	3.00	2.92	2.83	2.75	2.65
18	8.29	6.01	5.09	4.58	4.25	4.01	3.84	3.71	3.60	3.51	3.37	3.23	3.08	3.00	2.92	2.84	2.75	2.66	2.57
19	8.19	5.93	5.01	4.50	4.17	3.94	3.77	3.63	3.52	3.43	3.30	3.15	3.00	2.92	2.84	2.76	2.67	2.58	2.49
20	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46	3.37	3.23	3.09	2.94	2.86	2.78	2.69	2.61	2.52	2.42
21	8.02	5.78	4.87	4.37	4.04	3.81	3.64	3.51	3.40	3.31	3.17	3.03	2.88	2.80	2.72	2.64	2.55	2.46	2.36
22	7.95	5.72	4.82	4.31	3.99	3.76	3.59	3.45	3.35	3.26	3.12	2.98	2.83	2.75	2.67	2.58	2.50	2.40	2.31
23	7.88	5.66	4.76	4.26	3.94	3.71	3.54	3.41	3.30	3.21	3.07	2.93	2.78	2.70	2.62	2.54	2.45	2.35	2.26
24	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26	3.17	3.03	2.89	2.74	2.66	2.58	2.49	2.40	2.31	2.21
25	7.77	5.57	4.68	4.18	3.86	3.63	3.46	3.32	3.22	3.13	2.99	2.85	2.70	2.62	2.53	2.45	2.36	2.27	2.17
30	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07	2.98	2.84	2.70	2.55	2.47	2.39	2.30	2.21	2.11	2.01
40	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.89	2.80	2.66	2.52	2.37	2.29	2.20	2.11	2.02	1.92	1.80
60	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72	2.63	2.50	2.35	2.20	2.12	2.03	1.94	1.84	1.73	1.60
120	6.85	4.79	3.95	3.48	3.17	2.96	2.79	2.66	2.56	2.47	2.34	2.19	2.03	1.95	1.86	1.76	1.66	1.53	1.38
∞	6.63	4.61	3.78	3.32	3.02	2.80	2.64	2.51	2.41	2.32	2.18	2.04	1.88	1.79	1.70	1.59	1.47	1.32	1.00

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